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THE STABILITY PROBLEM OF COUPLED VIBRATIONS OF A BEAM WITH IMPERFECT ELASTIC CHARACTERISTICS AND DYNAMIC ABSORBERS

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Abstract. This study investigates the stability of coupled transverse vibrations of a beam with hysteresis-type elastic–dissipative characteristics and dynamic absorbers under harmonic excitation. The conditions for stability are expressed analytically depending on the system parameters, and the results of numerical analysis along with corresponding conclusions are presented. In particular, the variation of stability conditions depending on the elastic elements of the dynamic absorbers, as well as the changes in the stability conditions of steady-state vibrations of the considered system, are analyzed.

Introduction. A considerable number of scientific studies have been devoted to the problem of reducing vibrations of distributed-parameter systems using dynamic absorbers. In the work [1], the vibrations of a beam and a dynamic absorber attached to it were investigated. It was shown that, depending on the system parameters, the frequency may take values lower than, greater than, or equal to the partial frequency of the dynamic absorber. In the work [2], the coupled vibrations of a beam with two dynamic absorbers symmetrically positioned with respect to its ends were studied experimentally, and a comparative analysis of their oscillatory behavior was carried out.

It is well known that the investigation of vibrations of distributed-parameter mechanical systems coupled with dynamic absorbers possessing nonlinear elastic–dissipative characteristics represents a rather complex problem due to a number of factors. These factors significantly influence the nature of the system's vibrations. The governing equations of motion of such systems are nonlinear differential equations, and their solution requires the application of special mathematical methods. In the works [3,4], problems of nonlinear vibrations of a beam with installed dynamic absorbers under harmonic excitation were considered, taking into account hysteresis-type elastic–dissipative characteristics. The solutions were obtained in the form of transfer functions. In addition, the dynamics of nonlinear vibrations [5,6] and their stability properties [7,8] have also been investigated.

Based on these studies, the analysis of beam vibrations and their suppression processes remains a relevant and important problem.

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In the present paper, the stability of steady-state vibrations of a beam equipped with two dynamic absorbers is considered. Their motion is described by the differential equations presented in the work [9].

To solve the problem, the method of expanding the transverse vibrations of the beam into a series of mode shapes is employed. This approach is convenient for optimizing the parameters of dynamic absorbers for different types of beam vibrations under various boundary conditions, particularly when repeated computation of the amplitude–frequency characteristics of the system is required. The results of the works [9,10] indicate that if the damping decrement of the elastic–damping material of the dynamic absorber is sufficiently large, the nonlinearity of the internal resistance characteristics of the beam material does not significantly affect the beam vibrations or the determination of optimal parameters of the dynamic absorber. Therefore, it is assumed that the energy dissipation within the beam material follows the hypothesis of E.S. Sorokin [11]. A beam of length *l*, width *b*, and height *h* is rigidly attached to a vibrating base whose motion is specified along the *Oz* axis. Dynamic absorbers are installed at points with coordinates x_1 and x_2 along the beam (Fig. 1).

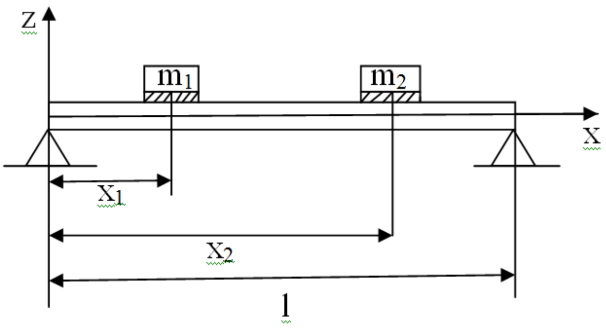

Fig. 1. A beam with installed dynamic absorbers

Materials and Methods. First, it is possible to drive the differential equations of motion for the coupled transverse vibrations of a beam with hysteresis-type elastic–dissipative characteristics and dynamic absorbers [4].

$$\frac{\partial^2 \mathrm{M}}{\partial \mathrm{x}^2}+\rho F\frac{\partial^2 w}{\partial t^2}-c_1R_1\delta_1(x-x_1)\zeta_1-c_2R_2\delta_2(x-x_2)\zeta_2=-\rho F\frac{\partial^2 w_0}{\partial t^2};$$

$$m_1\frac{\partial^2 w(x_1)}{\partial t^2}+m_1\frac{\partial^2\zeta_1}{\partial t^2}+c_1R_1\zeta_1=-m_1\frac{\partial^2 w_0}{\partial t^2}; \qquad (1)$$

$$m_2\frac{\partial^2 w(x_2)}{\partial t^2}+m_2\frac{\partial^2\zeta_2}{\partial t^2}+c_2R_2\zeta_2=-m_2\frac{\partial^2 w_0}{\partial t^2},$$

where M is the bending moment; ρ, F represent the density and cross-sectional area of the beam, respectively; w is the displacement of the beam; w_0 is the displacement of the base; $w(x_1), w(x_2)$ are the displacements of the points where the dynamic absorbers are attached; c_1, c_2 are the stiffness coefficients of the elastic elements of the dynamic absorbers; m_1, m_2 are the masses of the dynamic absorbers; ζ_1, ζ_2 denote the displacements of the dynamic absorbers relative to the beam; $\delta(x-x_1), \delta(x-x_2)$ are Dirac delta functions; x_1, x_2 are the locations where the dynamic absorbers are installed.

$$R_1 = 1 + (-\nu_1 + j\nu_2)[D_0 + f(\zeta_{1\text{от}})];$$
$$R_2 = 1 + (-\eta_1 + j\eta_2)[E_0 + g(\zeta_{2\text{от}})].\,, \quad (2)$$

$j^2 = -1$; $\nu_1, \nu_2, \eta_1, \eta_2$ are coefficients determined depending on the dissipative properties of the materials. $f(\zeta_{1n}), g(\zeta_{2n})$ represent the logarithmic decrements of the vibrations [13, 14].

$$f(\zeta_{1\text{от}}) = \sum_{\text{K}_1}^{r_1} D_{K_1} \zeta_{1\text{от}}^{\text{K}_1}; \quad (3)$$

$$g(\zeta_{2\text{от}}) = \sum_{\text{K}_2}^{r_2} E_{K_2} \zeta_{2\text{от}}^{\text{K}_2}, \quad (4)$$

$D_0, D_1, \ldots, D_{r_1}, E_0, E_2, \ldots, E_{r_2}$ are parameters dependent on the material properties of the elastic elements of the dynamic absorbers and are determined experimentally [6].

The relationship between the normal stress σ_N and the relative strain $\xi_{\text{от}}$ can be expressed as follows [5]:

$$\sigma_N = \text{E}[1 + j\eta_c]\xi_{nis}, \quad (5)$$

where E is the elastic modulus of the beam material; η_c is the damping coefficient, which is determined depending on the dissipative properties of the material.

For the relative strain (5), we write the following expression:

$$\xi_{nis} = \frac{\partial^2 w}{\partial x^2} z, \quad (6)$$

where z is the axis directed along the cross-section of the beam. Let's calculate the bending moment acting on the beam's cross-section [14].

$$M = 2b\int_0^{h/2} \sigma_{\text{н}} z dz = 2bE[1 + j\eta_c]\frac{\partial^2 w}{\partial x^2}\int_0^{h/2} z^2 dz = EJ[1 + j\eta_c]\frac{\partial^2 w}{\partial x^2}, \quad (7)$$

where $J = \frac{bh^3}{12}$ is the second moment of area (area moment of inertia); b is the width of the beam, and h is the height of the beam.

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Substituting the obtained expression for the bending moment into the first equation of system (1), we obtain the following system of differential equations:

$$EJ[1+j\eta_c]\frac{\partial^4 w}{\partial x^4}+\rho F\frac{\partial^2 w}{\partial t^2}-c_1R_1\delta_1(x-x_1)\zeta_1-c_2R_2\delta_2(x-x_2)\zeta_2 = -\rho F\frac{\partial^2 w_0}{\partial t^2};$$

$$m_1\frac{\partial^2 w(x_1)}{\partial t^2}+m_1\frac{\partial^2 \zeta_1}{\partial t^2}+c_1R_1\zeta_1=-m_1\frac{\partial^2 w_0}{\partial t^2}; \qquad (8)$$

$$m_2\frac{\partial^2 w(x_2)}{\partial t^2}+m_2\frac{\partial^2 \zeta_2}{\partial t^2}+c_2R_2\zeta_2=-m_2\frac{\partial^2 w_0}{\partial t^2}.$$

In solving the system of equations (8), it is possible to assume the displacement of the beam in the following form:

$$w(x,t)=\sum_{i=1}^{\infty}u_i(x)q_i(t). \qquad (9)$$

where $q_i(t)$ is a function of time; $u_i(x)$ is the mode shape (natural vibration form) of the beam, which satisfies the following equation:

$$EJ\frac{\partial^4 u_i}{\partial x^4}-\rho F p_i^2 u_i=0, \qquad (10)$$

where p_i is the natural frequency of the beam.

Substituting solution (9) into the system of equations (8) and taking into account relation (10), after performing the transformations, it is possible to obtain:

$$\ddot{q}_i+(1+j\eta_c)p_i^2q_i-\mu_1\mu_{0i}n_1^2u_{i1}R_1\zeta_1-\mu_2\mu_{0i}n_2^2u_{i2}R_2\zeta_2=-d_iW_0;$$

$$u_{i1}\ddot{q}_i+\ddot{\zeta}_1+n_1^2R_1\zeta_1=-W_0; \qquad (11)$$

$$u_{i2}\ddot{q}_i+\ddot{\zeta}_2+n_2^2R_2\zeta_2=-W_0.$$

$\mu_1=\frac{m_1}{m_c}$; $\mu_2=\frac{m_2}{m_c}$; $\mu_{0i}=\frac{1}{d_{2i}}$; $d_i=\frac{d_{1i}}{d_{2i}}$; $d_{1i}=\int_0^l u_i dx$; $d_{2i}=\int_0^l u_i^2 dx$; $m_c=\rho Fl$ is the mass of the beam.

For harmonic excitation, the base acceleration is taken in the following form:

$$W_0=w_0\cos\omega t,$$

where w_0 is the amplitude of the base acceleration and ω is the frequency.

The transfer functions of system (11) are determined in the following form:

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$$q(j\omega) = -\frac{B_1(\omega) + jB_2(\omega)}{B_3(\omega) + jB_4(\omega)} \cdot w_0;$$

$$\zeta_1(j\omega) = -\frac{B_5(\omega) + jB_6(\omega)}{B_3(\omega) + jB_4(\omega)} \cdot w_0;$$

$$\zeta_2(j\omega) = -\frac{B_7(\omega) + jB_8(\omega)}{B_3(\omega) + jB_4(\omega)} \cdot w_0;$$

where

$$B_1(\omega) = d_i\omega^4 - [T_1n_1^2(1-\nu_1N_1) + T_2n_2^2(1-\theta_1N_2)]\omega^2 +$$

$$+n_1^2n_2^2T_3[(1-\nu_1N_1)(1-\theta_1N_2) - \nu_2\theta_2N_1N_2];$$

$$B_2(\omega) = -[T_1n_1^2\nu_2N_1 + T_2n_2^2\theta_2N_2]\omega^2 + n_1^2n_2^2T_3[(1-\nu_1N_1)\theta_2N_2 +$$

$$+(1-\theta_1N_2)\nu_2N_1];$$

$$B_3(\omega) = -\omega^6 + [p_i^2 + n_1^2T_6(1-\nu_1N_1) + n_2^2T_7(1-\theta_1N_2)]\omega^4 -$$

$$-\{n_1^2p_i^2(1-\nu_1N_1-\eta_c\nu_2N_1) + n_2^2p_i^2(1-\theta_1N_2-\eta_c\theta_2N_2) +$$

$$+n_1^2n_2^2T_8[(1-\nu_1N_1)(1-\theta_1N_2) - \nu_2\theta_2N_1N_2]\}\omega^2 +$$

$$+n_1^2n_2^2[(1-\nu_1N_1)(1-\theta_1N_2) - \nu_2\theta_2N_1N_2 -$$

$$-\eta_c(1-\nu_1N_1)\theta_2N_2 - \eta_c(1-\theta_1N_2)\nu_2N_1];$$

$$B_4(\omega) = [\eta_cp_i^2 + n_1^2T_6\nu_2N_1 + n_2^2T_7\theta_2N_2]\omega^4 -$$

$$-\{n_1^2p_i^2[\eta_c(1-\nu_1N_1) + \nu_2N_1] +$$

$$+n_2^2p_i^2[\eta_c(1-\theta_1N_2) + \theta_2N_2] + n_1^2n_2^2T_8[(1-\nu_1N_1)\theta_2N_2 +$$

$$+(1-\theta_1N_2)\nu_2N_1]\}\omega^2 + n_1^2n_2^2[(1-\nu_1N_1)\theta_2N_2 + (1-\theta_1N_2)\nu_2N_1 +$$

$$+\eta_c(1-\nu_1N_1)(1-\theta_1N_2) - \eta_c\nu_2\theta_2N_1N_2];$$

$$B_5(\omega) = (1-d_iu_{i1})\omega^4 - [p_i^2 + T_4n_2^2(1-\theta_1N_2)]\omega^2 +$$

$$+n_2^2p_i^2[1-(\theta_1+\eta_c\theta_2)N_2];$$

$$B_6(\omega) = -(\eta_cp_i^2 + n_2^2T_4\theta_2N_2)\omega^2 + n_2^2p_i^2[\theta_2N_2 + \eta_c(1-\theta_1N_2)];$$

$$B_7(\omega) = (1-d_iu_{i2})\omega^4 - [p_i^2 + T_5n_1^2(1-\nu_1N_1)]\omega^2 +$$

$$+n_1^2p_i^2[1-(\nu_1+\eta_c\nu_2)N_1];$$

$$B_8(\omega) = -(\eta_cp_i^2 + n_1^2T_5\nu_2N_1)\omega^2 + n_1^2p_i^2[\nu_2N_1 + \eta_c(1-\nu_1N_1)];$$

$$T_1 = d_i + \mu_{0i}\mu_1u_{i1};\ T_2 = d_i + \mu_{0i}\mu_2u_{i2};\ T_3 = d_i + \mu_{0i}(\mu_1u_{i1} + \mu_2u_{i2});$$

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$T_4 = 1 + \mu_{0i}\mu_2 u_{i2}(u_{i2} - u_{i1}) - u_{i1}d_i$; $T_5 = 1 + \mu_{0i}\mu_1 u_{i1}(u_{i1} - u_{i2}) - u_{i2}d_i$;

$T_6 = 1 + \mu_{0i}\mu_1 u_{i1}^2$; $T_7 = 1 + \mu_{0i}\mu_2 u_{i2}^2$; $T_8 = 1 + \mu_{0i}(\mu_1 u_{i1}^2 + \mu_2 u_{i2}^2)$;

Let's seek the solutions of the obtained system of differential equations of motion (11) in the following form:

$$q_i = a_i \cos(\omega t + \alpha_i);$$
$$\zeta_1 = b_1 \cos(\omega t + \beta_1); \quad (12)$$
$$\zeta_2 = b_2 \cos(\omega t + \beta_2),$$

where $a_i, \alpha_i, b_1, \beta_1, b_2, \beta_2$ are slowly varying amplitude and phase functions [7].

By calculating the corresponding derivatives of the solutions (12), substituting them into the system of differential equations of motion (11), and taking into account the expression for the base acceleration, it is possible to obtain a system of equations in normal form corresponding to the system of differential equations of motion (11).

$$\dot{a}_i = -\frac{1}{2\omega}[-d_i w_0 \sin\alpha_i + \eta_c p_i^2 a_i - \mu_1\mu_{0i}n_1^2 u_{i1}b_1 H_1 - \mu_2\mu_{0i}n_2^2 u_{i2} b_2 sin\varphi_2];$$

$$\dot{\alpha}_i = -\frac{1}{2\omega a_i}[-d_i w_0 \cos\alpha_i + \left(\omega^2 - p_i^2\right)a_i + \mu_1\mu_{0i}n_1^2 u_{i1} b_1 H_2 + \mu_2\mu_{0i}n_2^2 u_{i2} b_2 cos\varphi_2];$$

$$\dot{b_1} = -\frac{1}{2\omega}[-(1 - d_i u_{i1})w_0 \sin\beta_1 + n_1^2\left(1 + \mu_1\mu_{0i}u_{i1}^2\right)\nu_2 N_1 b_1 - u_{i1}a_i p_i^2 H_3 + \mu_2\mu_{0i}n_2^2 u_{i1}u_{i2}b_2 sin\varphi_3];$$

$$(13)$$

$$\dot{\beta}_1 = -\frac{1}{2\omega b_1}\left[-(1 - d_i u_{i1})w_0 \cos\beta_1 + \left(\omega^2 - n_1^2\left(1 + \mu_1\mu_{0i}u_{i1}^2\right)(1 - \nu_1 N_1)\right) b_1 + u_{i1}a_i p_i^2 H_4 - \mu_2\mu_{0i}n_2^2 u_{i1}u_{i2}b_2 cos\varphi_3\right];$$

$$\dot{b_2} = -\frac{1}{2\omega}[-(1 - d_i u_{i2})w_0 \sin\beta_2 - u_{i2}a_i p_i^2 H_5 - \mu_1\mu_{0i}n_1^2 u_{i1}u_{i2}b_1 sin\varphi_3];$$

$$\dot{\beta}_2 = -\frac{1}{2\omega b_2}\left[-(1 - d_i u_{i2})w_0 \cos\beta_2 + \left(\omega^2 - n_2^2\left(1 + \mu_2\mu_{0i}u_{i2}^2\right)\right) b_2 + u_{i1}a_i p_i^2 H_6 - \mu_1\mu_{0i}n_1^2 u_{i1}u_{i2}b_1 cos\varphi_3\right],$$

where

$$H_1 = (1 - \nu_1 N_1)\sin\varphi_1 + \nu_2 N_1 \cos\varphi_1\,; H_2 = (1 - \nu_1 N_1)\cos\varphi_1 - \nu_2 N_1 \sin\varphi_1\,;$$
$$H_3 = \eta_c \cos\varphi_1 - \sin\varphi_1\,; H_4 = \eta_c \sin\varphi_1 + \cos\varphi_1\,; H_5 = \eta_c \cos\varphi_2 - \sin\varphi_2\,;$$
$$H_6 = \eta_c \sin\varphi_2 + \cos\varphi_2\,.\,\varphi_1 = \beta_1 - \alpha_i, \varphi_2 = \beta_2 - \alpha_i, \varphi_3 = \beta_2 - \beta_1.$$

To investigate the stability of the system's stationary vibrations, we use Lyapunov's first approximation method [12,13]. From the system of equations (13), using Lagrange's variational method, we obtain the following characteristic equation.

$$\lambda^6 + A_1\lambda^5 + A_2\lambda^4 + A_3\lambda^3 + A_4\lambda^2 + A_5\lambda + A_6 = 0, \qquad (14)$$

where λ is the characteristic value; $A_1, A_2, A_3, A_4, A_5, A_6$ are coefficients that depend on the system parameters.

The Hurwitz criterion takes the following form:

$$A_1 > 0, \qquad A_1A_2 - A_3 > 0, \; -A_1^2A_4 + A_1A_2A_3 + A_1A_5 - A_3^2 > 0,$$
$$A_1^2A_2A_6 - A_1^2A_4^2 - A_1A_2^2A_5 + A_1A_2A_3A_4 - A_1A_3A_6 + 2A_1A_4A_5$$
$$+A_2A_3A_5 - A_3^2A_4 - A_5^2 > 0,$$
$$-A_1^3A_6^2 + 2A_1^2A_2A_5A_6 + A_1^2A_3A_4A_6 - A_1^2A_4^2A_5 - A_1A_2^2A_5^2 -$$
$$-A_1A_2A_3^2A_6 + A_1A_2A_3A_4A_5 - 3A_1A_3A_5A_6 + 2A_1A_4A_5^2 -$$
$$-A_2A_3A_5^2 + A_3^3A_6 - A_3^2A_4A_5 - A_5^3 > 0, \qquad (15)$$
$$(-A_1^3A_6^2 + 2A_1^2A_2A_5A_6 + A_1^2A_3A_4A_6 - A_1^2A_4^2A_5 - A_1A_2^2A_5^2 -$$
$$-A_1A_2A_3^2A_6 + A_1A_2A_3A_4A_5 - 3A_1A_3A_5A_6 + 2A_1A_4A_5^2 +$$
$$+A_2A_3A_5^2 + A_3^3A_6 - A_3^2A_4A_5 - A_5^3)A_6 > 0.$$

Numerical calculations and results. For the numerical analysis, we consider a beam made of 40X grade steel with the following dimensions:

$l = 25 \cdot 10^{-2}\, m; b = 10^{-2}\, m; h = 2 \cdot 10^{-3}\, m,$ the mechanical damping coefficient is $\eta_c = 0.85;$ the masses and coordinates of the dynamic absorbers are: $m_1 = m_2 = 0.0027335\, kg;\; x_1 = \frac{l}{3}, x_2 = \frac{2l}{3}$. The coefficients [15–18]:

$$\nu_2 N_1 = 817684.6 \cdot a_{1*}^2 - 1.920207 \cdot 10^{12} \cdot a_{1*}^4 + 1.556859 \cdot 10^{18} \cdot a_{1*}^6.$$

The graphs of the coefficients A_1-A_6, depending on the variation of the stiffness of the elastic elements of the dynamic absorbers, are presented below.

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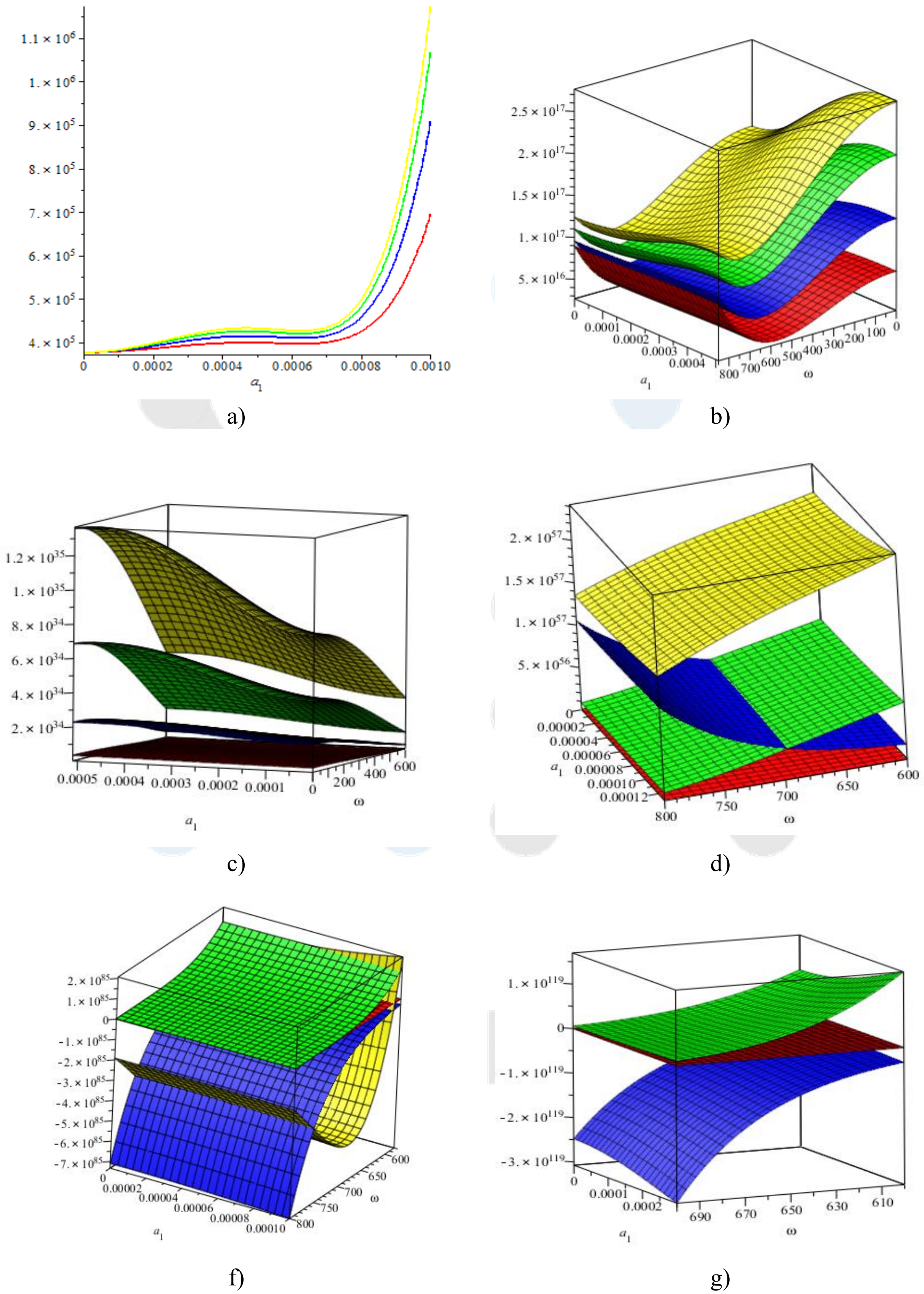

Figure 2. Graphs of the coefficients A_1-A_6 as functions of the variation in the stiffness of the elastic elements of the dynamic absorbers.

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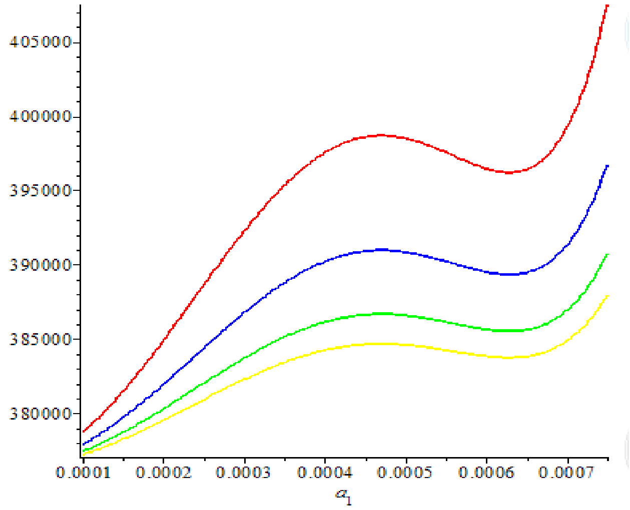
As can be seen from Fig. 2, when the stiffness $c_1 = c_2 = 600$ increases from 600 to 1500 N/m, the final coefficients remain positive in the range where the stiffness is approximately up to 1000 N/m. After that, they decrease and become negative. This indicates the possibility of the occurrence of unstable vibrations in the system [19–22].

Now, let us examine how these coefficients vary depending on the mass ratios μ_1 and μ_2.



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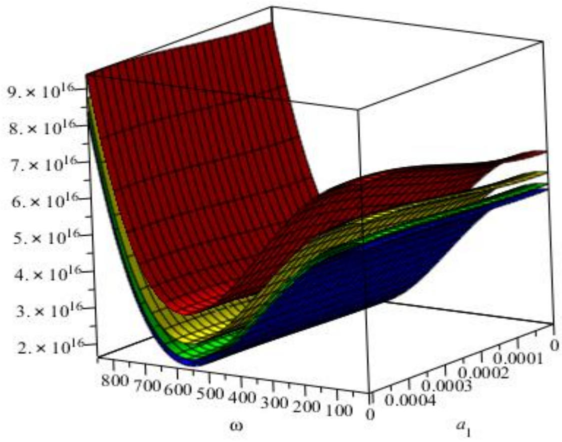

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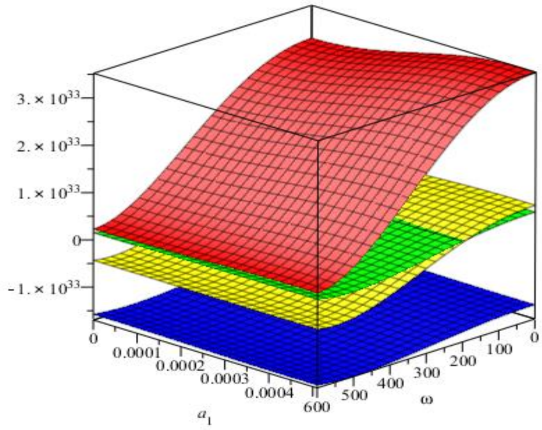

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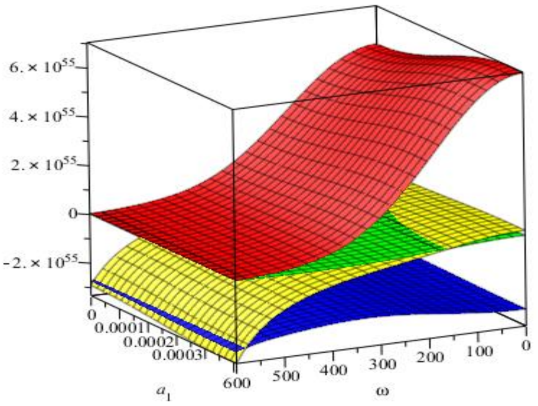

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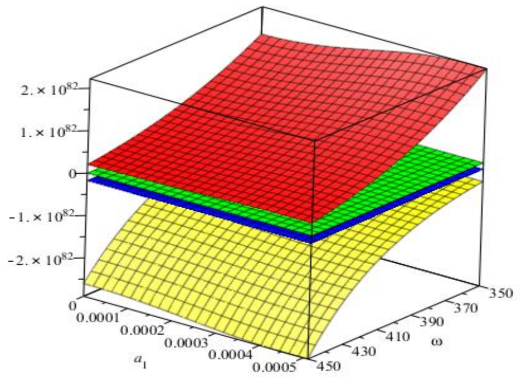

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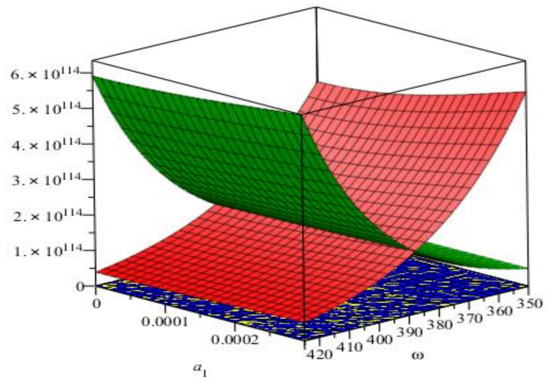

g)

Figure 3. Graphs of the coefficients A_1-A_6 as functions of the variation in the mass ratios of the dynamic absorbers to the beam.

As shown in Fig. 3, the variation of these coefficients is illustrated. The presented graphs correspond to the following values: $\mu_1 = \mu_2 = 0.04$ (red), 0.07 (green), 0.12 (blue), and 0.18 (yellow).

From the graphs, it can be observed that the initial coefficients take positive values in almost all cases, whereas the subsequent coefficients attain negative values around μ=0.12. This indicates the possibility of the existence of unstable solutions in the system.

Conclusion. In this study, the stability of the amplitudes of stationary harmonic vibrations of a beam equipped with dynamic absorbers having hysteresis-type elastic–dissipative characteristics was investigated. The stability conditions were determined in analytical form and analyzed based on numerical calculations. The parameter ranges of the system corresponding to stable and unstable motions were identified.

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