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INFLUENCEOF THE THREE-BODY COULOMB EFFECTS ON THE VALUES OF ASYMPTOTIC NORMALIZATION COEFFICIENTS FOR ^{22}Ne+p→^{23}Na

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Absract

The contribution of the three-body Coulomb dynamics of the transfer mechanism to the partial wave amplitudes at $l \gg 1$ for the peripheral proton-transfer reaction ^{22}Ne(^{3}He,d)^{23}Na is estimated within the three-body approach combining the dispersion method and the DWBA approach. For this reaction, the Coulomb renormalization factors, arising owing to correctly taking into account the three-body Coulomb dynamics of the proton-transfer mechanism in the DWBA cross sections, are calculated. A new estimate is obtained for the values of the asymptotic normalization coefficients for ^{22}Ne+p→^{23}Na, which also have astrophysical interest.

Key words: Asymptotic normalization coefficient, DWBA, differential cross section, transfer reaction, Coulomb renormalization factor.

Introduction

It is believed that, owing to the peripheral character of the radiative capture reactions at stellar energies [1], the overall normalization of the direct astrophysical *S* factors can be correctly determined through the asymptotic normalization coefficients (ANC) of the overlap function of the bound-state wave functions of the entrance and final nuclei for α particle or proton removed from the residual nucleus. Using modified DWBA approach, the ANC value for ^{22}Ne+p→^{23}Na configuration was found from the peripheral proton-transfer reaction ^{22}Ne(^{3}He,d)^{23}Na in [2]. The modified DWBA approach developed in [3] on the basis of the idea proposed in [4] is restricted only to the zeroth- and first-order perturbation approach in the optical Coulomb polarization potential V_i^C (or V_f^C) in the transition operator sandwiched by the entrance- and exit-channel wave functions, respectively. However, in reality, the power expansion in $V_{f,i}^C$ series for the Coulomb part of the total three body transition operator for the considered peripheral transfer reactions contains higher order terms (the second and higher) in $V_{f,i}^C$ [5]. So the account of these expansion terms in the modified DWBA calculations is necessary [6] to verify an accuracy of the traditional "post" form of DWBA used in [2] for obtaining the ANC values. It should be emphasized that the task of obtaining the ANC values with high precision is especially important in nuclear astrophysics for calculation of the direct one-charged-particle capture ^{22}Ne(p,γ)^{23}Na reaction. The aim of this present work is to verify the reliability of the modified-DWBA-approach predictions of the available ANC values for ^{22}Ne+p→^{23}Na of astrophysical interest [2]. The purpose of this work is to study the influence of the three-body Coulomb dynamics of the transfer mechanism on the DWBA cross-section calculations for the peripheral proton-transfer ^{22}Ne(^{3}He,d)^{23}Na reaction. The consideration is based on the three-body approach combining the dispersion method and DWBA approach proposed in [8].

2. Generalization of the Modified DWBA Approach

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The cross section in the conventional modified DWBA [3] takes the following form:

$$\frac{d\sigma^{DWBA}}{d\Omega}(E_i,\theta) = \sum \mathrm{C}^2_{ay;l_xj_x}\,\mathrm{C}^2_{Aa;l_Bj_B}\,\frac{\sigma^{\mathrm{DWBA}}_{l_xj_xl_Bj_B}(E_i,\theta)}{\mathrm{b}^2_{ay;l_xj_x}\mathrm{b}^2_{Aa;l_Bj_B}} \qquad (1)$$

where $\mathrm{C}^2_{ay;l_xj_x}$ and $\mathrm{C}^2_{Aa;l_Bj_B}$ are the ANCs for the x →y + *a* and A + *a* → B vertices and $\sigma^{\mathrm{DWBA}}_{l_xj_xl_Bj_B}(E_i,\theta)$ is the single-particle DWBA cross section. $\mathrm{b}_{ay;l_xj_x}$ and $\mathrm{b}_{Aa;l_Bj_B}$ are single particle ANCs.

The part of the amplitude $M^{TB}(E_i,\cos\theta)$ which involves not only the sum of the zeroth and the first terms of the expansion of the transition operator in the series in $\Delta V^C{}_{f,i}$ but all terms of such expansion in $\Delta V^C{}_{f,i}$ can be represented in the form

$$M^{TB}(E_i,\cos\theta) \simeq M^{TBDW}(E_i,\cos\theta) = M^{DWBA}(E_i,\cos\theta) + \Delta M^{DW}(E_i,\cos\theta) \qquad (2)$$

where

$$\Delta M^{DW}(E_i,\cos\theta) = \sum \langle \chi^{(-)}{}_{k_f} I_{Aa} \,|\, \Delta V^C{}_f G_C\, \Delta V^C{}_i \,|\, I_{ay}\chi^{(+)}{}_{k_i}\rangle \qquad (3)$$

The presence of G_C in the second term $\Delta M^{DW}(E_i,\cos\theta)$ of M^{TBDW} (1) corresponds to the account of the three-body Coulomb dynamics of the transfer mechanism, which involves all possible subsequent Coulomb rescattering of three (*A*, *a*, and *y*) particles in the intermediate state, in the DWBA amplitude [6,9]. The part of the $\Delta M^{DW}(E_i,\cos\theta)$ amplitude corresponding to the $V^C{}_{yA}G_CV^C{}_{Aa}$ term in the transition operator is plotted in diagram c in the figure.

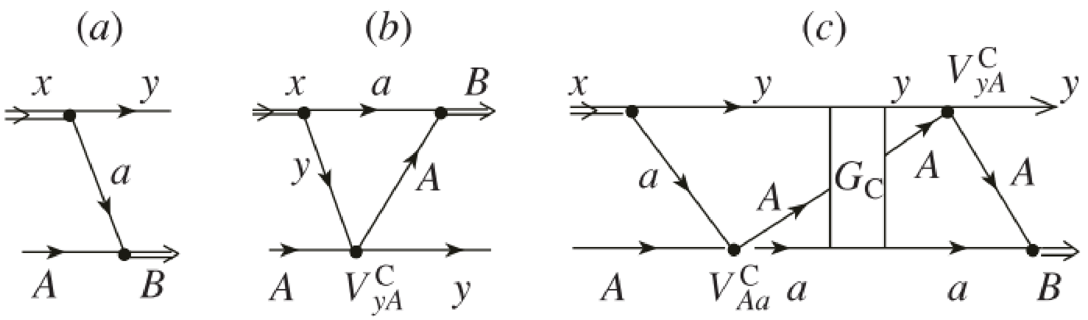

Fig.1. Diagrams describing transfer of particle *a* and taking into account possible subsequent Coulomb rescattering of particles *A, y* and *a* in the intermediate state

But the presence of the nuclear distortions in the entrance and exit channels in the amplitude (3) leads also to the appearance in $M^{TBDW}(E_i,\cos\theta)$ of the terms regular at cos θ = ξ with the singularities located further from the physical region than ξ. Therefore, the amplitude (3) can be considered as a generalization of the standard DWBA amplitude in which both the optical Coulomb–nuclear distortions in the entrance and exit channels and the three-body Coulomb dynamics of transfer mechanism in the intermediate state are taken into account. Then the cross section for the peripheral transfer reaction *A(x,y)B* with correct account of the three-body Coulomb effects in the transfer mechanism has the form

$$\frac{d\sigma^{TBDW}}{d\Omega}(E_i,\theta) = \sum \mathrm{C}^2_{ay;l_xj_x}\,\mathrm{C}^2_{Aa;l_Bj_B}\,\frac{\sigma^{\mathrm{TBDW}}_{l_xj_xl_Bj_B}(E_i,\theta)}{\mathrm{b}^2_{ay;l_xj_x}\mathrm{b}^2_{Aa;l_Bj_B}} \qquad (4)$$

Here, it should be emphasized that the presence of the term $\Delta V^C{}_f G_C \Delta V^C{}_i$ in the transition operator for the $\sigma^{\mathrm{TBDW}}_{l_xj_xl_Bj_B}(E_i,\theta)$ leads to considerable complication in cross-section calculation.

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The explicit forms have been found for the behavior of the exact amplitude $M^{TB}(E_i, cos\ \theta)$ [9] (or $M^{TBDW}(E_i, cos\ \theta)$) and of the DWBA amplitude $M^{DWBA}(E_i, cos\ \theta)$ [6, 9] (see also [10]) in the vicinity of the nearest singularity point cos θ = ξ on the basis of the three-body Faddeev equations for the Alt–Grassberger–Sandhas operators and within the sub-Coulomb three-body DWBA approach, respectively. They have the following forms:

$$M_l^{DWBA}(E_i) \simeq D\ \tilde{N}^{DWBA} \frac{e^{-l\ ln\ \tau}}{l^{\frac{1}{2}+\eta_x+\eta_B-i(\eta_i+\eta_f)}},\ l \gg 1 \qquad (5)$$

$$M^{TB}{}_l(E_i) \simeq D\ \tilde{N}\ \frac{e^{-l\ ln\ \tau}}{l^{\frac{1}{2}+\eta_x+\eta_B-i(\eta_i+\eta_f)}},\ l \gg 1 \qquad (6)$$

where

$$D = \pi^{3/2} \frac{m_a\ exp[i\pi(l_x + l_B + \eta_x + \eta_B)/2]}{\mu_{Aa}\mu_{ay}k_i k_f \sqrt{\tau^2 - 1}}$$
$$\times\ (\xi^2 - 1)^{[1+\eta_x+\eta_B-i(\eta_i+\eta_f)]/2} C_{ay;l_x j_x} C_{Aa;l_B j_B},$$

with $\tilde{N}^{DWBA} = \frac{N^{DW}}{\Gamma(1+\eta_x+\eta_B-i(\eta_i+\eta_f))}$, $\tilde{N} = \frac{N}{\Gamma(1+\eta_x+\eta_B-i(\eta_i+\eta_f))}$, $\tau = \xi + \sqrt{\xi^2 - 1}$, and η_i (η_f) is the Coulomb parameter in the entrance (exit) channel. It should also be noted that the explicit expression of the peripheral partial wave amplitudes $M_{post;\,l}^{DWBA}(E_i)$ for $l \gg 1$ corresponding to the "post" approximation of the DWBA amplitude [6] has the same form as Eq. (4), but the factor of $\tilde{N}^{DWBA}$ in it can be replaced by that of $\tilde{N}_{post}^{DWBA} = N_{post}^{DW}/\Gamma(1+\eta_x+\eta_B-i(\eta_i+\eta_f))$. The explicit expressions of the Coulomb renormalization factors N_{post}^{DW}, N^{DW} and N are given by Eqs. (14), (26), and (27) of [6], respectively. As seen from (5) and (6), the peripheral partial amplitudes $M_l^{DWBA}(E_i)$ and $M_l^{TB}(E_i)$ at $l \gg 1$ have the same dependence on l. Their ratio has the form

$$R = \left|\frac{M_l^{TB}(E_i)}{M_l^{DWBA}(E_i)}\right| = \left|\frac{\tilde{N}}{\tilde{N}^{DWBA}}\right| = \left|\frac{N}{N^{DW}}\right| \qquad (7)$$

for $l \gg 1$. Analogously, the ratio of the peripheral partial amplitudes $M_l^{DWBA}(E_i)$ and $M_{post;\,l}^{DWBA}(E_i)$ has the form

$$\tilde{R} = \left|\frac{M_l^{DWBA}(E_i)}{M_{post;\,l}^{DWBA}(E_i)}\right| = \left|\frac{\tilde{N}^{DWBA}}{\tilde{N}_{post}^{DWBA}}\right| = \left|\frac{N^{Dw}}{N_{post}^{DW}}\right| \qquad (8)$$

for $l \gg 1$. Thus, the influence of the three-body Coulomb effects on the peripheral partial amplitudes of the exact, "post" form, and "post" approximation amplitudes is different. So, for the peripheral transfer reaction *A(x,y)B*, owing to the fact that the main contribution comes from the peripheral partial amplitudes, the cross section (4) near the stripping peak ($\theta \approx \theta_{peak}$) can be given, as is proposed in [10], by

$$\frac{d\sigma^{TBDW}}{d\Omega}(E_i, \theta) = R^2 \frac{d\sigma^{DWBA}}{d\Omega}(E_i, \theta) \qquad (8)$$

Here, $d\sigma^{DWBA}/d\Omega$ is given by Eq. (1) and the value of R allows us to estimate the contribution of the term $\Delta M^{DW}(E_i, cos\ \theta)$ to the DWBA cross section in the stripping peak and, consequently, to the values of ANC, $C_{ay;lxjx}{}^2 C_{Aa;lBjB}{}^2$, obtained within the modified DWBA approach in [2] for astrophysical application.

3. Analysis of the peripheral proton transfer reaction $^{22}Ne(^3He,d)^{23}Na$

First, we analyze the $^{22}Ne(^3He,d)^{23}Na$ reaction. The DWBA cross-section calculations performed in [2] at incident ^3He-ion energy of 20 MeV showed that this reaction is strongly

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peripheral. Furthermore, they are related to the "nondramatic" case. Therefore, Eq. (8) can be used to test the accuracy of the ANC values for ^{22}Ne + p → ^{23}Na extracted from the analysis of the ^{22}Ne(^{3}He,d)^{23}Na reaction from [2].

The results of calculation of the Coulomb renormalization factor R^2 for the peripheral ^{22}Ne(^{3}He,d)^{23}Na reaction are presented in the fifth column of the table, respectively. As is seen from the table, the contribution of the three-body Coulomb effects determined by the values of R^2 to the DWBA cross-section calculations performed in [2] for this peripheral reaction at least near the main peak in the angular distribution varies from 9.0 to 9.2% depending on the energy level (the ground and first three excited states) of the residual nucleus of ^{23}Na. The figures in the parentheses in the fifth columns of the table correspond to the renormalization factor $\tilde{R}$, and the comparison of $|N_{post}^{DW}|^2$ and $|N^{DW}|^2$ allows one to estimate the extent to which $V^C{}_{yA} - V^C{}_f$ in the transition operator (9) influences the value $|N^{DW}|^2$. The calculations show that the difference between $|N^{DW}|^2$ and $|N|^2$ for the aforesaid reactions is smaller than that between $|N_{post}^{DW}|^2$ and $|N^{DW}|^2$ (compare also R^2 and $\tilde{R}^2$). Such a situation usually occurs in the "nondramatic" case [6] and, so, Eq. (8) is correct for this reaction.

One can also see from the table that the influence of the three-body Coulomb dynamics on the DWBA cross sections increases when the residual nucleus ^{23}Na in the reaction under consideration is formed in excited states, and the contribution of the three-body Coulomb effects into the DWBA cross-section calculations can exceed the errors of the experimental cross sections.

Table. ANC values for ^{22}Ne + p → ^{23}Na and the Coulomb renormalization factors R^2 and $\tilde{R}^2$ for the ^{22}Ne(^{3}He,d)^{23}Na reaction

E_x, MeV	J^π	nl_j	$C^2_{Aa;l_Bj_B}$, fm^{-1} [2]	R^2 $(\tilde{R}^2)$	$C^2_{Aa;l_Bj_B}$, fm^{-1} present
0.0	$3/2^+$	$1d_{3/2}$	3.84±0.25	1.090 (1.191)	3.52±0.229
0.44	$5/2^+$	$1d_{5/2}$	21.99±0.64	1.090 (1.196)	20.174±0.587
2.392	$1/2^+$	$2s_{1/2}$	77.44±2.56	1.091 (1.232)	70.980±2.346
2.982	$3/2^+$	$1d_{3/2}$	6.71±0.76	1.092 (1.249)	6.144±0.695

4. Conclusion

The Coulomb renormalization factor, which is given by the ratio of the peripheral partial wave amplitudes of the exact amplitude $M^{TB}(E_i, cos\ \theta)$ (or the same for $M^{TBDW}(E_i, cos\ \theta)$) to those of the DWBA amplitude $M^{DWBA}(E_i, cos\ \theta)$ [6, 10] and determines exactly the contribution of the three-body Coulomb dynamics of the proton transfer mechanism to the DWBA cross section in the main peak, is calculated for the peripheral ^{22}Ne(^{3}He,d)^{23}Na reaction at incident ^{3}He-ion energy of 20 MeV.

It is shown that the contribution of the three-body Coulomb dynamics of the proton-transfer mechanism to the DWBA cross sections (Δ%) for the ^{22}Ne(^{3}He,d)^{23}Na reaction leading to the ground and first three excited states of ^{23}Na and it is verified from Δ = 9.0% till Δ = 9.2% as a function of the energy level of ^{23}Na, and the value of Δ increases as the residual nucleus ^{23}Na is formed in the weakly bound excited states being of astrophysical interest. New estimates have been obtained for the ANC values for p + ^{22}Ne → ^{23}Na, where the ^{23}Na nucleus is formed in the ground and three excited states.

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