ISSN: 2582-4686 SJIF 2021-3.261,SJIF 2022-2.889, 2024-6.875 ResearchBib IF: 10.25 / 2025

THE MULTIDISCIPLINARY JOURNAL OF SCIENCE AND TECHNOLOGY

VOLUME-6, ISSUE-3

MATHEMATICAL AND STATISTICAL ANALYSIS OF THE IDEAL AND REAL MOTION EQUATIONS OF A PROJECTILE LAUNCHED AT AN ANGLE TO THE HORIZON

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Annotation: This article presents scientific, theoretical, and practical information regarding the equations of motion and the trajectory of a projectile launched at an angle α to the horizon with an initial velocity V0, for ideal flight (projectile motion is considered without taking air resistance into account) and real flight (projectile motion is considered taking into account the projectile's mass and air resistance). Furthermore, the article provides a mathematical and statistical analysis of the projectile's maximum altitude above the ground and its flight range (geographical range) as a function of the flight angle to the horizon, presented in tabular form.

Keywords: Projectile, horizontal, vertical, ideal and real conditions, trajectory, zenith angle, mathematical statistics.

During artillery operations, the probability that a projectile fired at an angle will accurately strike its target depends on a number of factors. This relationship has been confirmed both in military training practice and within the laws of mechanics. If the influence of air resistance on a projectile's trajectory—namely, its maximum height and maximum range—is neglected, such motion is referred to as the *ideal trajectory* of the projectile. In this case, the projectile is subjected only to the force of gravity.

To derive the equations of motion for a projectile launched with an initial velocity v_0 in the XOY plane, the initial velocity vector must be resolved into its components along the coordinate axes (see Figure 1).

$$\begin{cases} v_x = v_0 \sin\alpha \\ v_x = v_0 \cos\alpha - st \end{cases} \text{ from this } \begin{cases} x(t) = v_0 t \sin\alpha \\ y(t) = v_0 \sin\alpha - \dfrac{gt^2}{2} \end{cases} \quad (1)$$

From the first equation, we eliminate the time variable t, that is, we remove t from the system. Solving the first equation for, we obtain $t = \dfrac{x}{v_0 \cos\alpha}$. Substituting this expression into the second equation, we derive $y(x) = v_0 \frac{x}{\cos\alpha}\sin\alpha - \frac{gx^2}{2v_0^2\cos^2\alpha}$ and thereby obtain the complete trajectory equation

The second equation represents the trajectory of a projectile launched at an angle a(alpha), that is, the equation of the curve traced by the projectile in space. From this relationship, it follows that the trajectory is a parabola.

To determine the maximum height reached by the projectile relative to the horizontal line, we note that at the highest point the vertical velocity is $v_y = 0$. Using this condition, from $v_{0\sin\alpha} = gt$ we obtain the time $t = \frac{v_0 \sin\alpha}{g}$ Substituting this time value into the vertical motion equation $y(t)$, we derive the third equation.

$$H = y(t) v_0 \sin\alpha \frac{v_0 \sin\alpha}{g} - \frac{g}{2}\frac{v_0^2 \sin\alpha}{g^2} = \frac{v_0^2 \sin^2\alpha}{g} - \frac{v_0^2 \sin^2\alpha}{2g} = \frac{v_0^2 \sin^2\alpha}{g} \quad (3)$$

The range of the projectile is also considered one of the fundamental parameters of external ballistics. In determining the range, the total flight time t is taken as twice the time required for the projectile to reach its maximum height, since points located symmetrically with respect to the vertex of the parabola have equal values. In other words, if the projectile moves along a parabolic trajectory, the ascent time is equal to the descent time. Taking this into account, the total flight time is $t = 2t = \frac{2v_0^2 \sin\alpha}{g}$

the horizontal motion equation of the projectile is

$$S_x = v_x t = v_0 \cos\alpha \frac{2v_0 \sin\alpha}{g} = \frac{2v_0^2 \sin\alpha \cos\alpha}{g} = \frac{v_0^2 \sin 2\alpha}{g} \quad (4)$$

which has been established. The resultant velocity of the projectile is

$$v = \sqrt{v_0^2 \cos^2\alpha + v_0^2 \sin^2\alpha + g^2 t^2 - 2v_0 gt \sin\alpha} = \sqrt{2v_0^2 - 2v_0 gt \sin\alpha + g^2 t^2}$$

The angular coefficient of the projectile's velocity vector can be expressed through the relation $tg\alpha = \frac{v_y}{v_x} = \frac{v_0 \sin\alpha^{-st}}{v_0 \cos\alpha}$ (4) as shown in the fourth formula [1,3].

By varying the firing angle α of a projectile launched with an initial velocity V_0, it is possible to demonstrate that the projectile's range (its trajectory length) changes accordingly. For this purpose, we fire the projectile at an initial velocity of $v_0 = 600\frac{m}{s}$ first at angles smaller than 45° ,and later at angles larger than the obtained results can be observed in table 1 below.

Table 1

(α), grad	10°	20°	30°	45°	55°	65°	75°
$x_{c,km}$	12,313	23,140	31,177	36,000	33,829	27,578	18,000

ISSN: 2582-4686 SJIF 2021-3.261,SJIF 2022-2.889, 2024-6.875 ResearchBib IF: 10.25 / 2025

THE MULTIDISCIPLINARY JOURNAL OF SCIENCE AND TECHNOLOGY

VOLUME-6, ISSUE-3

Practical results have shown that, under identical conditions and using identical weapons, the probability of projectiles hitting a target—when launched at a constant initial velocity v_0 – varies significantly depending on the launch angle. These differences are illustrated in Table 1. The conclusion is that the trajectory and flight range of a projectile with mass m and initial velocity v_0 – depend solely on the $\sin 2\alpha$ of the launch angle confirming the formula:

$_{2\,0}v \ \sin 2\, x\, g \ =$

Therefore, to strike a nearby target, the launch angle must be either less than or greater than 45°. To hit the farthest possible target, the projectile must be launched at an angle of exactly 45°. The projectiles are launched at an angle relative to the horizontal with an initial velocity v_0.

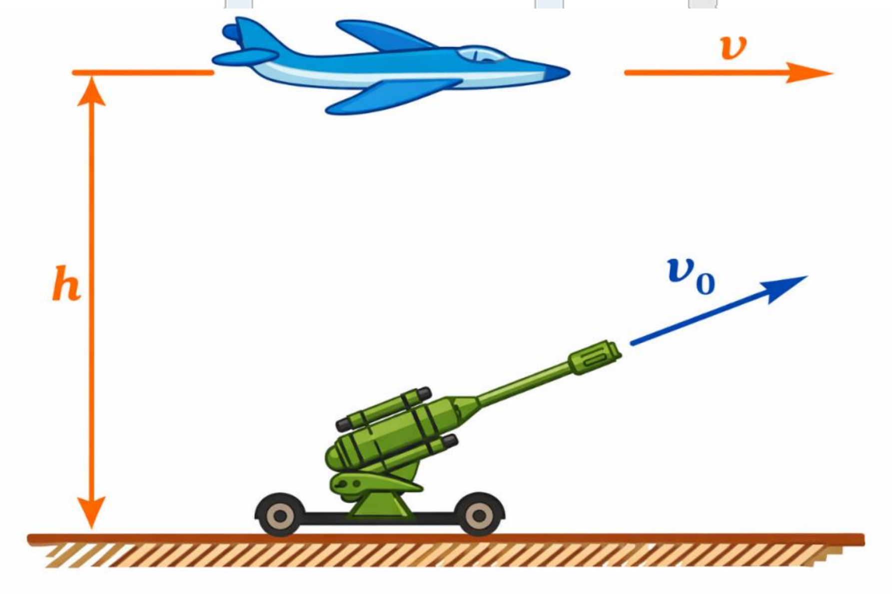
Derive the equation of motion for the projectile, assuming that air resistance is proportional to velocity.To solve this problem, refer to Figure 1. Use the projections of the initial velocity v_0 along the OX and OY axes.

Based on the conditions of the projectile's motion:"

According to the fundamental law of dynamics: 0000, we formulate the differential equation of motion with respect to the Ox axis.

$x_0 = 0 \qquad t = 0 \qquad y_0 = 0 \qquad v_x = v_0 \cos\alpha \qquad v_y = v_0 \sin\alpha$

$v_x = v_0 \cos\alpha; \qquad v_y = v_0 \sin\alpha - gt$



$$C_1 = \ln v_x = \ln v_0 \cos\alpha; \qquad \ln v_x = -\frac{k}{m}t + \ln v_0 \cos\alpha = \ln e^{-\frac{k}{m}t} +$$

$$+\ln v_0 \cos\alpha = \ln v_0 \cos\alpha e^{-\frac{k}{m}t}$$

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$$v_x = v_0 \cos\alpha e^{-\frac{k}{m}t} \qquad v_x = \frac{dx}{dt} = -\frac{mv_0}{k}\cos\alpha \int e^{-\frac{k}{m}t} d(-\frac{k}{m}t)$$

$$x(t) = -\frac{m}{k} v_0 \cos\alpha e^{-\frac{k}{m}t} + C_2 \qquad C_2 = \frac{m}{k} v_0 \cos\alpha$$

$$x(t) = -\frac{m}{k} v_0 \cos\alpha e^{-\frac{k}{m}t} + \frac{m}{k} v_0 \cos\alpha = \frac{m}{k} v_0 \cos\alpha (1 - e^{-\frac{k}{m}t}) \tag{5}$$

In conclusion, the ideal horizontal motion of a projectile (bullet) fired from a weapon follows the equation given in formula (5). At the same time, because the projectile undergoes a complex motion, it simultaneously follows the functions $x(t)$ and $y(t)$. In the vertical direction, the motion is described $y(t)$, and in the horizontal direction by $x(t)$. In this process, both gravitational force and air resistance are taken into account, that is…

$$m\frac{dv_y}{dt} = -kv_y - mg \tag{6}$$

$$m\frac{dv_y}{dt} = -kv_y - mg; \qquad dv_y = \frac{1}{m}\left(-kv_y - my\right)dt$$

$$x(0) = 0 \qquad y(0) = 0 \qquad v_x = v_0 \cos\alpha; \qquad v_y = v_0 \sin\alpha$$

$$\frac{dv_y}{kv_y + mg} = -\frac{1}{m}dt \tag{7}$$

$$\int \frac{dv_y}{kv_y + mg} = -\frac{1}{m}\int dt: \qquad \frac{1}{k}\int \frac{d\left(kv_y + mg\right)}{kv_y + mg} = -\frac{1}{m}t + C_1$$

$$\ln\left|kv_y + mg\right| = -\frac{k}{m}t + \ln\left|kv_0 \sin\alpha + mg\right|$$

$$\ln\left|kv_y + mg\right| = \ln e^{-\frac{k}{m}t} + \ln\left|kv_0 \sin\alpha + mg\right|$$

$$kv_y + mg = e^{-\frac{k}{m}t}\left(kv_0 \sin\alpha + mg\right)$$

$$v_y = e^{-\frac{k}{m}t}\left(kv_0 \sin\alpha + mg\right)\cdot\frac{1}{k} - \frac{mg}{k}$$

$$v_y = dy = e^{-\frac{k}{m}t}\left(kv_0 \sin\alpha + mg\right)\cdot\frac{1}{k}dt - \frac{mg}{k}dt$$

$$v_y = \frac{dy}{dt}$$

$$y(t) = -\left(kv_0 \sin\alpha + mg\right)\cdot\frac{1}{k}\cdot\frac{m}{k}e^{-\frac{k}{m}t} + \frac{mgt}{k} + C_2$$

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$$y(0) = -\frac{m}{k^2}(kv_0\sin\alpha + mg)e^{-\frac{k}{m}0} + C_2; \qquad C_2 = \frac{m}{k^2}(kv_0\sin\alpha + mg)$$

$$y(t) = \frac{m}{k^2}(kv_0\sin\alpha + mg) - \frac{m}{k^2}(kv_0\sin\alpha + mg)e^{-\frac{k}{m}t} + \frac{mgt}{k}$$

$$y(t) = \frac{m}{k^2}(kv_0\sin\alpha + mg)\left[1 - e^{-\frac{k}{m}t}\right] + \frac{mgt}{k} \qquad va \qquad x(t) = \frac{v_0 m\cos\alpha}{k}\left(1 - e^{-\frac{k}{m}t}\right) \tag{8}$$

When a projectile is fired from any type of weapon, the air resistance k, together with the initial velocity v_0 and the launch angle α relative to the horizontal plane, determines the projectile's real (non-ideal) motion. Under these conditions, the ballistic equations of the projectile can attain high accuracy only through differential equations. One of the main tasks in external ballistics (the kinematic problem) is the determination of the trajectory. To solve this problem, the following differential equation must be addressed:

$$m\frac{dv}{dt} = -kv_x \qquad \ln v_x = -\frac{k}{m}t + C_1$$

$$\frac{dv_x}{v_x} = -\frac{k}{m}dt \qquad C_1 = \ln v_x = \ln v_x\cos\alpha$$

$$\ln v_x = -\frac{k}{m}t + \ln v_0\cos\alpha = \ln e^{-\frac{k}{m}t} + \ln v_0\cos\alpha$$

$$v_x = v_0\cos\alpha e^{-\frac{k}{m}t}$$

$$dx = v_0\cos\alpha\int e^{-\frac{k}{m}t}dt$$

$$x = \frac{v_0}{k}\cos\alpha\int e^{-\frac{k}{m}t}d\left(-\frac{k}{m}t\right)$$

$$x(t) = -\frac{m}{k}v_0\cos\alpha e^{-\frac{k}{m}t} + C_2$$

$$\frac{m}{k}v_0\cos\alpha = C_2$$

$$x(t) = -\frac{m}{k}v_0\cos\alpha e^{-\frac{k}{m}t} + \frac{m}{k}v_0\cos\alpha = \frac{m}{k}v_0\cos\alpha\left(1 - e^{-\frac{k}{m}t}\right) \tag{9}$$

The real equation of motion of the projectile with respect to the horizontal axis has been derived through formula (9) (see Figure 1b).

In conclusion, whether a projectile (bullet) is fired at an angle in the ideal case—where air resistance is neglected—or in the real case, its trajectory remains a parabola whose branches are directed toward the ground. The real-motion equations differ from the ideal ones due to their complexity and the violation of trajectory symmetry. It has been demonstrated that, for projectiles fired with the same initial velocity, the range depends on the launch angle as well as the geographic latitude of the firing location.

ISSN: 2582-4686 SJIF 2021-3.261,SJIF 2022-2.889, 2024-6.875 ResearchBib IF: 10.25 / 2025

VOLUME-6, ISSUE-3

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