: 2582-4686 SJIF 2021-3.261,SJIF 2022-2.889, 2024-
5 ResearchBib IF: 8.848 / 2024
THE MULTIDISCIPLINARY JOURNAL OF SCIENCE AND TECHNOLOGY
VOLUME-4, ISSUE-10

DERIVATION OF THE WIENER-HOPF INTEGRAL EQUATION

Mirsaburova Gulbaxor Miraxmatovna
Tashkent state pedagogical University named after Nizami, Tashkent, Uzbekistan

Abstract. The problem with Bitsadze-Samarskii conditions on the boundary of ellipticity and a segment of the degeneracy line and the displacement condition on pieces of the boundary characteristics of the Gelleristedt equation with a singular coefficient is investigated. The uniqueness of the solution to the problem is proved using the maximum principle, and the existence of the solution is proved using the method of integral equations.

Keywords: singular coefficient, uniqueness of the solution to the problem, Wiener-Hopf equation, equation index.

1. INTRODUCTION

Using the theory of the singular integral equation and the theory of the Wiener-Hopf equation, the obtained system of equations reduced to the second-order Fredholm integral equation, the unique solution of which follows from the uniqueness of the solution to the given problem.

$$\tau_1(x)-\lambda\int_{c_1}^{1}\left(\frac{x-c_1}{t-c_1}\right)^{1-2\beta}\left(\frac{1}{t-x}-\frac{a}{1-xt}\right)\tau_1(t)dt=g_1(x),\qquad x\in(c_1,1),\tag{1}$$

$$g_1(x)=\lambda a k^{2\beta}\int_{c_1}^{1}\left(\frac{x-c_1}{t-c_1}\right)^{1-2\beta}\frac{\tau_1(t)dt}{x-q(t)}-\lambda b k^{1-2\beta}\int_{c_1}^{1}\left(\frac{x-c_1}{t-c_1}\right)^{1-2\beta}\frac{\tau_1(t)dt}{t-q(x)}+$$
$$+L^*[\tau_1]+F_3(x),\quad x\in(c_1,1).\tag{2}$$

The kernel of the first integral operator from (2) at point $(x,t)=(c_1,c_1)$ (where $q(c_1)=c_1$) has an isolated first-order singularity, so this operator is singled out separately. In (1), we assume that $c_1\neq 1$; if $c=1$, then problem TB turns into a Tricomi problem with a condition set at all points of characteristics AC, and the integral operators of the right-hand side (1) are identically equal to zero. In this case, a well-studied singular integral Tricomi equation is obtained.

2. PRELIMINARIES

Lemma. If $g_1(x)$ satisfies the Hölder condition for $x\in(c_1,1)$ and $g_1(x)\in L_p(c_1,1),\quad p>1,$ then the solution to equation (2) in the class of functions H, in which function $(x-c_1)^{2\beta-1}\tau_1(x)$ is limited for $x=1$ and can be limited for $x=c_1$, is expressed by the following formula:

THE MULTIDISCIPLINARY JOURNAL OF SCIENCE AND TECHNOLOGY
VOLUME-4, ISSUE-10

$$\tau_1(x)=\cos^2(\alpha\pi)g_0(x)+\frac{\sin(2\alpha\pi)}{2\pi}\int_{c_1}^{1}\left(\frac{x-c_1}{t-c_1}\right)^{3\alpha}\left(\frac{1-x}{1-t}\right)^{2\alpha}\left(\frac{1-c_1t}{1-c_1x}\right)^{\alpha}\times$$

$$\times\left(\frac{1}{t-x}-\frac{a}{1-xt}\right)g_1(t)dt,\qquad x\in(c_1,1). \tag{3}$$

Now, substituting the expression for $g_1(x)$ from (2) to (3) and separating the singular part of the equation, we obtain

$$\tau_1(x)=\lambda ak^{1-4\beta}\cos^2(\alpha\pi)\int_{c_1}^{1}\left(\frac{x-c_1}{t-c_1}\right)^{4\alpha}\frac{\tau_1(t)dt}{x-q(t)}-$$

$$-\lambda bk^{4\alpha}\cos^2(\alpha\pi)\int_{c_1}^{1}\left(\frac{x-c_1}{t-c_1}\right)^{4\alpha}\frac{\tau_1(t)dt}{t-q(x)}+$$

$$+\frac{\lambda bk^{1-4\alpha}\sin(2\alpha\pi)}{2\pi}\int_{c_1}^{1}\frac{(x-c_1)^{3\alpha}(1-x)^{2\alpha}}{(s-c_1)^{4\alpha}}A(x,s)\tau_1(s)ds-$$

$$-\frac{\lambda bk^{4\alpha}\sin(2\alpha\pi)}{2\pi}\int_{c_1}^{1}\frac{(x-c_1)^{3\alpha}(1-x)^{2\alpha}}{(s-c_1)^{4\alpha}}B(x,s)\tau_1(s)ds+$$

$$+R_1[\tau_1]+M_1(x),\qquad x\in(c_1,1), \tag{4}$$

where

$$A(x,s)=\int_{c_1}^{1}\frac{(t-c_1)^{\alpha}}{(1-t)^{2\alpha}}\left(\frac{1}{t-x}-\frac{a^{-1}}{1-xt}\right)\frac{dt}{t-q(s)} \tag{5}$$

$$B(x,s)=\int_{c_1}^{1}\frac{(t-c_1)^{\alpha}}{(1-t)^{2\alpha}}\left(\frac{1}{t-x}-\frac{a^{-1}}{1-xt}\right)\frac{dt}{s-q(t)}; \tag{6}$$

$$R_1[\tau_1]=\cos^2(\alpha\pi)L^*[\tau_1]+$$

$$+\frac{\sin(2\alpha\pi)}{2\pi}\int_{c_1}^{1}\left(\frac{x-c_1}{t-c_1}\right)^{3\alpha}\left(\frac{1-x}{1-t}\right)^{2\alpha}\left(\frac{1-c_1t}{1-c_1x}\right)^{\alpha}\left(\frac{1}{t-x}-\frac{a^{-1}}{1-xt}\right)L^*[\tau_1]dt+$$

$$+\frac{\lambda ak^{1-4\alpha}\sin(2\alpha\pi)}{2\pi}(x-c_1)^{3\alpha}(1-x)^{2\alpha}\int_{c_1}^{1}\frac{\tau_1(s)ds}{(s-c_1)^{4\alpha}}\times$$

$$\times\int_{c_1}^{1}\frac{(t-c_1)^{\alpha}}{(1-t)^{2\alpha}}\left(\frac{1}{t-x}-\frac{a^{-1}}{1-xt}\right)\left[\left(\frac{1-c_1t}{1-c_1x}\right)^{\alpha}-1\right]\frac{dt}{t-q(s)}-$$

$$-\frac{\lambda bk^{4\alpha}\sin(2\alpha\pi)}{2\pi}(x-c_1)^{3\alpha}(1-x)^{2\alpha}\int_{c_1}^{1}\frac{\tau_1(s)ds}{(s-c_1)^{4\alpha}}\times$$

THE MULTIDISCIPLINARY JOURNAL OF SCIENCE AND TECHNOLOGY
VOLUME-4, ISSUE-10

$$\times\int_{c_1}^{1}\frac{(t-c_1)^{\alpha}}{(1-t)^{\alpha}}\left[\left(\frac{1-c_1t}{1-c_1x}\right)^{\alpha}-1\right]\left(\frac{1}{t-x}-\frac{a^{-1}}{1-xt}\right)\frac{dt}{s-q(t)} \quad (7)$$

- is the regular operator,

$$M_1(x)=\cos^2(\alpha\pi)F_3(x)+$$

$$+\frac{\sin(2\alpha\pi)}{2\pi}\int_{c_1}^{1}\left(\frac{x-c_1}{t-c_1}\right)^{3\alpha}\left(\frac{1-x}{1-t}\right)^{2\alpha}\left(\frac{1-c_1t}{1-c_1x}\right)^{\alpha}\left(\frac{1}{t-x}-\frac{a^{-1}}{1-xt}\right)F_3(t)dt \quad (8)$$

– is a well-known function.

3. MAIR RESULTS

Theorem.To isolate in the third and fourth integrals (5) and (6), the kernels $\frac{1}{x-q(s)}$ and $\frac{1}{s-q(x)}$, which at points $(x,s)=(c_1,c_1)$ have isolated first-order singularities, we calculate $A(x,s)$ and $B(x,s)$ from (5) and (6).

Proof. The rational part of integral expressions (5) and (6) is decomposed into simple fractions:

$$\left(\frac{1}{t-x}-\frac{a^{-1}}{1-xt}\right)\frac{1}{t-q(s)}=\frac{1}{x-q(s)}\left(\frac{1}{t-x}-\frac{1}{t-q(s)}\right)-$$

$$-\frac{a^{-1}}{1-xq(s)}\left(\frac{1}{1-xt}+\frac{1}{t-q(s)}\right)$$

$$\left(\frac{1}{t-x}-\frac{a^{-1}}{1-xt}\right)\frac{1}{s-q(t)}=\frac{1}{s-q(x)}\left(\frac{1}{t-x}-\frac{k}{s-q(t)}\right)-$$

$$-\frac{a^{-1}}{k+x(s-\rho)}\left(\frac{x}{1-xt}+\frac{k}{s-q(t)}\right). \quad (9)$$

Considering decompositions (9), the right-hand parts of (5) and (6) are transformed to the following form:

$$A(x,s)=\frac{1}{x-q(s)}\big(J_1(x)-J_1(s)\big)-\frac{a^{-1}}{1-xq(s)}\big(xJ_3(x)+J_2(s)\big), \quad (10)$$

$$B(x,s)=\frac{1}{s-q(x)}\big(J_1(x)-kJ_4(s)\big)-\frac{a^{-1}}{k+x(s-\rho)}\big(xJ_3(x)+kJ_4(s)\big), \quad (11)$$

where

1. $$J_1(x)=\int_{c_1}^{1}\frac{(t-c_1)^{\alpha}}{(1-t)^{2\alpha}}\frac{dt}{t-x}=-\pi ctg\alpha\pi\frac{(x-c_1)^{\alpha}}{(1-x)^{2\alpha}}+$$

: 2582-4686 SJIF 2021-3.261,SJIF 2022-2.889, 2024-
5 ResearchBib IF: 8.848 / 2024

THE MULTIDISCIPLINARY JOURNAL OF SCIENCE AND TECHNOLOGY

VOLUME-4, ISSUE-10

$$+\frac{\Gamma(\alpha)\Gamma(1-2\alpha)}{\Gamma(1-\alpha)}\frac{(1-c_1)^{\alpha}}{(1-x)^{2\alpha}}F\left(-\alpha,1-2\alpha,1-\alpha;\frac{x-c_1}{1-c_1}\right); \qquad (12)$$

2.
$$J_2(s)=\int_{c_1}^{1}\frac{(t-c_1)^{\alpha}}{(1-t)^{2\alpha}}\frac{dt}{t-q(s)}=$$

$$=\frac{\Gamma(1-2\alpha)\Gamma(1+\alpha)}{\Gamma(2-\alpha)}\frac{(1-c_1)^{1-\alpha}}{1-q(s)}F\left(1-2\alpha,1,2-\alpha;\frac{1-c_1}{1-q(s)}\right); \qquad (13)$$

3.
$$J_3(s)=\int_{c1}^{1}\frac{(t-c_1)^{\alpha}}{(1-t)^{2\alpha}}\frac{dt}{1-xt}=$$

$$=\frac{\Gamma(1+\alpha)\Gamma(1-2\alpha)}{\Gamma(2-\alpha)}\frac{(1-c)^{1-\alpha}}{(1-c_1x)^{1-2\alpha}(1-x)^{2\alpha}}F\left(1-2\alpha,1-\alpha,2-\alpha;\frac{(1-c_1)x}{1-c_1x}\right); (14)$$

4.

$$J_4(s)=\int_{c_1}^{1}\frac{(t-c_1)^{\alpha}}{(1-t)^{2\alpha}}\frac{dt}{s-q(t)}=$$

$$=\frac{\Gamma(1+\alpha)\Gamma(1-2\alpha)}{\Gamma(2-\alpha)}\frac{(1-c_1)^{1-\alpha}}{s-c_1+k(1-c_1)}F\left(1-2\alpha,1,2-\alpha;\frac{k(1-c_1)}{s-c_1+k(1-c_1)}\right). \qquad (15)$$

Let us prove formula (12):

$$J_1(x)=\int_{c_1}^{1}\frac{(t-c_1)^{\alpha}}{(1-t)^{2\alpha}}\frac{dt}{t-x}=\lim_{\delta\to 0}\left[-\int_{c_1}^{x}\frac{(t-c_1)^{\alpha}}{(1-t)^{2\alpha}}\frac{dt}{(x-t)^{1-\delta}}+\int_{x}^{1}\frac{(t-c_1)^{\alpha}}{(1-t)^{2\alpha}}\frac{dt}{(t-x)^{1-\delta}}\right].$$

Here in the first and second integrals, by making the substitution of the variable integration $t=c_1+(x-c_1)\sigma$ and $t=1-(1-x)\sigma$, then using the integral representation of the hypergeometric Gauss function [1], we have

$$J_1(x)=\lim_{\delta\to 0}\left[-\frac{\Gamma(1+\alpha)\Gamma(\delta)}{\Gamma(1+\alpha+\delta)}\frac{(x-c_1)^{\alpha+\delta}}{(1-c_1)^{2\alpha}}F\left(1+\alpha,2\alpha,1+\alpha+\delta;\frac{x-c_1}{1-c_1}\right)+\right.$$

$$\left.+\frac{\Gamma(1-2\alpha)\Gamma(\delta)}{\Gamma(1-2\alpha+\delta)}\frac{(1-c_1)^{\alpha}}{(1-x)^{2\alpha-\delta}}F\left(1-2\alpha,-\alpha,1-2\alpha+\delta;\frac{1-x}{1-c_1}\right)\right]. \qquad (16)$$

Now, applying the auto-transformation formulas to the first and second terms of the right-hand part of Eq. (16) [1, p.10] and Bolz [1, p.11] *Smirnov M.M. Equations of mixed type. M.: Higher School. -1985. –304 p.*, we obtain:

$$J_1(x)=\lim_{\delta\to 0}\left\{\Gamma(\delta)\left[\frac{\Gamma(-\alpha-\delta)}{\Gamma(-\alpha)}-\frac{\Gamma(1+\alpha)}{\Gamma(1+\alpha+\delta)}\right]\times\right.$$

$$\times\frac{(1-c_1)^{-\delta}(x-c_1)^{\alpha+\delta}}{(1-x)^{2\alpha-\delta}}F\left(\delta,1-\alpha+\delta,1+\alpha+\delta;\frac{x-c_1}{1-c_1}\right)+$$

$$+\frac{\Gamma(1-2\alpha)\Gamma(\alpha+\delta)}{\Gamma(1-\alpha+\delta)}\frac{(1-c_1)^{\alpha}}{(1-x)^{2\alpha-\delta}}F\left(1-2\alpha,-\alpha,1-2\alpha+\delta;\frac{x-c_1}{1-c_1}\right)\Bigg\}. \quad (17)$$

In (17) moving to the limit as $\delta\to 0,$ considering equality

$$\lim_{\delta\to 0}\Gamma(\delta)\left[\frac{\Gamma(-\alpha-\delta)}{\Gamma(-\alpha)}-\frac{\Gamma(1+\alpha)}{\Gamma(1+\alpha+\delta)}\right]=-\pi ctg\alpha\pi$$

we get formula (12).

To prove formula (13), it is necessary to replace variable integration $t=1-(1-c_1)\sigma$ and then use the integral representation of the hypergeometric function.

When proving formula (14), the replacement of variable integration $t=c_1+(1-c_1)\sigma$ done, and then the auto transformation formula is applied to the resulting hypergeometric function. To prove formula (15), the replacement of variable integration $t=c_1+(1-c_1)\sigma$ is done, and then the formula is applied to the resulting hypergeometric function $F(a,b,c;z)=(1-z)^{-b}F\left(c-a,b,c;\frac{z}{z-1}\right).$

Considering formulas (12) - (15), equalities (5) and (6) are written in the following form:

$$A(x,s)=\frac{1}{x-q(s)}\left[-\pi ctg(\alpha\pi)\frac{(x-c_1)^{\alpha}}{(1-x)^{2\alpha}}+A_0(x,s)\right]+A_1(x,s), \quad (18)$$

$$B(x,s)=\frac{1}{s-q(x)}\left[-\pi ctg(\alpha\pi)\frac{(x-c_1)^{\alpha}}{(1-x)^{2\alpha}}+B_0(x,s)\right]+B_1(x,s). \quad (19)$$

Here:

$$A_0(x,s)=\frac{\Gamma(\alpha)\Gamma(1-2\alpha)}{\Gamma(1-\alpha)}\frac{(1-c_1)^{\alpha}}{(1-x)^{2\alpha}}F\left(-\alpha,1-2\alpha,1-\alpha;\frac{x-c_1}{1-c_1}\right)-$$

$$-\frac{\Gamma(1-2\alpha)\Gamma(1+\alpha)}{\Gamma(2-\alpha)}\frac{(1-c_1)^{1-\alpha}}{1-q(s)}F\left(1-2\alpha,1,2-\alpha;\frac{1-c_1}{1-q(s)}\right), \quad (20)$$

$$A_1(x,s)=-\frac{a}{1-xq(s)}\left(xJ_3(x)+J_2(s)\right);$$

$$B_0(x,s)=\frac{\Gamma(\alpha)\Gamma(1-2\alpha)}{\Gamma(1-\alpha)}\frac{(1-c_1)^{\alpha}}{(1-x)^{2\alpha}}F\left(-\alpha,1-2\alpha,1-\alpha;\frac{x-c_1}{1-c_1}\right)-$$

$$-\frac{k\Gamma(1+\alpha)\Gamma(1-2\alpha)}{\Gamma(2-\alpha)}\frac{(1-c_1)^{1-\alpha}}{s-c_1+k(1-c_1)}F\left(1-2\alpha,1,2-\alpha;\frac{k(1-c_1)}{s-c_1+k(1-c_1)}\right), \quad (21)$$

$$B_1(x,s) = -\frac{a}{k + x(s-\rho)}\left(xJ_3(x) + kJ_4(s)\right).$$

Substituting expressions $A(x,s)$ and $B(x,s)$ from (18) and (19) in equality (4), considering identities $\frac{\sin(2\alpha\pi)}{2\pi}\pi\, ctg(\alpha\pi) = \cos^2(\alpha\pi)$, we obtain:

$$\tau_1(x) - \frac{\lambda\sin(2\alpha\pi)}{2\pi}\int_{c_1}^{1}\frac{(x-c_1)^{3\alpha}(1-x)^{2\alpha}}{(s-c_1)^{4\alpha}}\left[\frac{ak^{1-4\alpha}A_0(x,s)}{x-q(s)} - \frac{bk^{4\alpha}B_0(x,s)}{s-q(x)} + \right.$$

$$\left. + ak^{1-4\alpha}A_1(x,s) - bk^{4\alpha}B_1(x,s)\right]\tau_1(s)ds = R_1[\tau_1] + M_1(x), \quad x \in (c_1,1), \tag{22}$$

where $A_1(x,s)$, $B_1(x,s)$ are the regular kernels.

We will find estimates for $A_0(x,s)$ and $B_0(x,s)$ in the vicinity of points $x = c_1$, $s = c_1$. For the hyperactive geometric function $F(1-2\alpha,1,2-\alpha;z)$ in (20) and (21), applying the Bolts formula, we transform them to the following form:

$$A_0(x,s) = \frac{\Gamma(\alpha)\Gamma(1-2\alpha)}{\Gamma(1-\alpha)}\frac{(1-c_1)^{\alpha}}{(1-x)^{2\alpha}}F\left(-\alpha,1-2\alpha,1-\alpha;\frac{x-c_1}{1-c_1}\right) -$$

$$-\frac{\Gamma(\alpha)\Gamma(1-2\alpha)}{\Gamma(1-\alpha)}\frac{(1-c_1)^{1-\alpha}}{1-q(s)}F\left(1-2\alpha,1,1-\alpha;\frac{c_1-q(s)}{1-q(s)}\right) -$$

$$-\frac{\Gamma(-\alpha)\Gamma(1+\alpha)}{\Gamma(1)}\left(\frac{1-c_1}{1-q(s)}\right)^{1-\alpha}\left(\frac{c_1-q(s)}{1-q(s)}\right)^{\alpha}F\left(1+\alpha,1-\alpha,1+\alpha;\frac{c_1-q(s)}{1-q(s)}\right), \tag{23}$$

$$B_0(x,s) = \frac{\Gamma(\alpha)\Gamma(1-2\alpha)}{\Gamma(1-\alpha)}\frac{(1-c_1)^{\alpha}}{(1-x)^{2\alpha}}F\left(-\alpha,1-2\alpha,1-\alpha;\frac{x-c_1}{1-c_1}\right) -$$

$$-\frac{k\Gamma(-\alpha)\Gamma(1-2\alpha)}{\Gamma(1-\alpha)}F\left(1-2\alpha,1,1-\alpha;\frac{s-c_1}{s-c_1+k(1-c_1)}\right) -$$

$$-\frac{k\Gamma(-\alpha)\Gamma(1+\alpha)}{\Gamma(1)}\frac{(1-c_1)^{1-\alpha}}{s-c_1+k(1-c_1)}\left(\frac{s-c_1}{s-c_1+k(1-c_1)}\right)^{\alpha}\times$$

$$\times F\left(1+\alpha,1-\alpha,1+\alpha;\frac{s-c_1}{s-c_1+k(1-c_1)}\right). \tag{24}$$

In (23) and (24), applying the following formula to the hypergeometric function $F(1,1-2\alpha,1-\alpha;z)$

$$F(a,b,c;z) = (1-z)^{-b}F\left(c-a,b,c;\frac{z}{z-1}\right),$$

we obtain:

$$A_0(x,s)=\frac{\pi k^{\alpha}(s-c_1)^{\alpha}}{\sin(\alpha\pi)(1-q(s))^{2\alpha}}+(x-q(s))A_2(x,s), \tag{25}$$

$$B_0(x,s)=\frac{\pi k^{\alpha}(s-c_1)^{\alpha}}{\sin(\alpha\pi)(s-c_1+k(1-c_1)^{2\alpha}}+(s-q(x))B_2(x,s), \tag{26}$$

where

$$A_2(x,s)=\frac{\Gamma(\alpha)\Gamma(1-2\alpha)}{\Gamma(1-\alpha)(1-c_1)^{\alpha}(x-q(s))}\left[\left(1-\frac{x-c_1}{1-c_1}\right)^{-2\alpha}F\left(-\alpha,1-2\alpha,1-\alpha;\frac{x-c_1}{1-c_1}\right)-\right.$$
$$\left.-\left(1-\frac{q(s)-c_1}{1-c_1}\right)^{-2\alpha}F\left(-\alpha,1-2\alpha,1-\alpha;\frac{q(s)-c_1}{1-c_1}\right)\right], \tag{27}$$

$$B_2(x,s)=\frac{\Gamma(\alpha)\Gamma(1-2\alpha)}{\Gamma(1-\alpha)(1-c_1)^{\alpha}(s-q(x))}\left[\left(1-\frac{x-c_1}{1-c_1}\right)^{-2\alpha}F\left(-\alpha,1-2\alpha,1-\alpha;\frac{x-c_1}{1-c_1}\right)-\right.$$
$$\left.-\left(1-\frac{c_1-s}{k(1-c_1)}\right)^{-2\alpha}F\left(-\alpha,1-2\alpha,1-\alpha;\frac{c_1-s}{k(1-c_1)}\right)\right]. \tag{28}$$

Now from (27) and (28), considering the following identities

$$\frac{x-c_1}{1-c_1}-\frac{q(s)-c_1}{1-c_1}=\frac{x-q(s)}{1-c_1};\quad \frac{x-c_1}{1-c_1}-\frac{c_1-s}{k(1-c_1)}=\frac{s-q(x)}{k(1-c_1)},$$

by virtue of the Lagrange formula: $f(b)-f(a)=f'(c)(b-a)$, it is easy to make sure that in the vicinity of points $(x,s)=(c_1,c_1)$ functions $A_2(x,s)=O(1)$, $B_2(x,s)=O(1)$ are limited.

Considering the representations (25) and (26), equation (22) is transformed to the following form:

$$\tau_1(x)=\lambda\cos(\alpha\pi)\int_{c_1}^{1}\left(\frac{x-c_1}{s-c_1}\right)^{3\alpha}\left(\frac{ak^{1-3\alpha}}{x-q(s)}-\frac{bk^{3\alpha}}{s-q(x)}\right)\tau_1(s)ds+$$
$$+R_2[\tau_1]+M_1(x),\ x\in(c_1,1), \tag{29}$$

where

$$R_2[\tau_1]=R_1[\tau_1]+\frac{\lambda\sin(2\alpha\pi)}{2\pi}\int_{c_1}^{1}\frac{(x-c_1)^{3\alpha}(1-x)^{2\alpha}}{(s-c_1)^{4\alpha}}\times$$
$$\times\left[ak^{1-4\alpha}\left(A_1(x,s)+A_2(x,s)\right)-bk^{4\alpha}\left(B_1(x,s)+B_2(x,s)\right)\right]\times$$

$$\times\tau_1(x)ds+\lambda\cos(\alpha\pi)\int_{c_1}^{1}\left(\frac{x-c_1}{s-c_1}\right)^{3\alpha}\left\{ak^{1-3\alpha}\left[\left(\frac{1-x}{1-q(s)}\right)^{2\alpha}-1\right]\times\right.$$

$$\left.\times\frac{1}{x-q(s)}-bk^{5\alpha}\left[\left(\frac{1-x}{s-c_1+k(1-c_1)}\right)^{2\alpha}-\left(\frac{1}{k}\right)^{2\alpha}\right]\frac{1}{x-q(x)}\right\}\tau_1(s)ds \qquad (30)$$

- is the regular kernel.

By swapping variables *([2] Mirsaburov M., Ruziev M. On a boundary value problem for a class of mixed-type equations in an unbounded domain. //Differential equations. 2011: -vol. 47. - No. 1. pp. 112-119)* $x=c_1+(1-c_1)e^{-y}$, $s=c_1+(1-c_1)e^{-t}$ in equation (29) and using the notation $\rho(y)=\tau_1[c_1+(1-c_1)e^{-y}]e$, we obtain the Wiener-Hopf equation *[3] Gakhov F.D., Chersky Yu.I. Convolution type equations. // Main editorial office of the Physics and Mathematics Lit., Moscow 1978. –p.269., [4] Eleev V.A. On some boundary value problems for a degenerate second-order hyperbolic equation. // Differential equations. -1976. -vol. 12. -No. 1. -pp. 46-58.*

$$\rho(y)=\int_0^{\infty}K(y-t)\rho(t)dt+R_2[\rho]+M_2(y), \qquad (31)$$

where $R_2(\rho)=e^{(3\alpha-1/2)y}R_1[\tau_1]-$ is the regular operator, $M_2(y)=e^{(3\alpha-1/2)y}M_1(c_1+(1-c_1)e^{-y})-$ is a well-known function,

$$K(x)=\lambda\cos(\alpha\pi)\left[\frac{ak^{1-3\alpha}}{ke^{x/2}+e^{-x/2}}-\frac{bk^{3\alpha}}{e^{x/2}+ke^{-x/2}}\right].$$

Kernel $K(x)$ of equation (31) is continuously differentiable and has the exponential order of decreasing at infinity. Note that by virtue of condition, $\beta_0>(1-m)/3$ the value of $(3\alpha-1/2)$ is negative, then operator $R_2[\rho]$ and function $M_2(y)$ also have the exponential order of decreasing at infinity. Fredholm's theorems *[5] Salakhitdinov M.S. Mathematics physics the equations. Toshkent: Fan. -2002. –b.448.* for integral equations of the convolution type are fulfilled only for one partial case, namely when the index of these equations is zero.

The index of equation (31) is the index of expression $1-K^{\wedge}(x)$ *[6, p. 56] Volkodavov V.F. On the uniqueness of the solution of the TN problem for one equation of mixed type. // Volga Mathematical collection of the Kuibyshev State Pedagogical Institute. -1970.-No. 1. -pp. 55-65,* taken with the opposite sign: Now using the following formula

$$\int_{-\infty}^{+\infty}\frac{e^{-ixt}dt}{ke^{t/2}+e^{-t/2}}=\frac{\pi e^{ix\ln k}}{\sqrt{k}\,ch(\pi x)},$$

we have

$$K^{\wedge}(x)=\frac{\lambda\cos(\alpha\pi)(ak^{1-3\alpha}e^{ix\ln k}-bk^{3\alpha}e^{-ix\ln k})}{\sqrt{k}ch(\pi x)}. \quad (32)$$

$$\left|\frac{\lambda\cos(\alpha\pi)(ak^{1-3\alpha}+bk^{3\alpha})}{\sqrt{k}}\right|<1 \quad (33)$$

From the representation (32), by virtue of (33), it follows that $\left|\operatorname{Re}K^{\wedge}(x)\right|<1$; it is also obvious that $\operatorname{Re}K^{\wedge}(x)=O(1/ch(\pi x))$ for large enough $|x|$.

Hence, we conclude that

$$Ind(1-K^{\wedge}(x))=\frac{1}{2\pi}\left[arctg\frac{\operatorname{Im}(1-K^{\wedge}(x))}{\operatorname{Re}(1-K^{\wedge}(x))}\right]\Bigg|_{-\infty}^{+\infty}=0,$$

that is, changing the argument of expression $1-K^{\wedge}(x)$ on the real axis, represented in complete revolutions, *Volkodavov V.F. On the uniqueness of the solution of the TN problem for one equation of mixed type. // Volga Mathematical collection of the Kuibyshev State Pedagogical Institute. -1970.- No. 1. -pp. 55-65.* Hence, the uniqueness of the solution to the problem implies the unambiguous solvability of equation (31), and, therefore, of problem TB.

REFERENCES

1. Смирнов М.М. Уравнения смешанного типа. М.: Высшая школа. -1985. – с.304.

2. Мирсабуров М., Рузиев М. Об одной краевой задаче для одного класса уравнений смешанного типа в неограниченной области. //Дифференциальные уравнения. -2011: -т. 47. -№ 1. -С. 112-119.

3. Гахов Ф.Д., Черский Ю.И. Уравнения типа свертки. // Главная редакция физико-математической лит., Москва -1978. –с.269.

4. Елеев В.А. О некоторых краевых задачах для одного вырождающегося гиперболического уравнения второго порядка. // Дифференциальные уравнения. -1976. -т. 12. -№ 1. -С. 46-58.

5. Салахитдинов М.С. Математик физика тенгламалари. Тошкент: Фан. -2002. –б.448.

6. Волкодавов В.Ф. О единственности решения задачи TN для одного уравнения смешанного типа. // Волжский математический сборник Куйбышевского гос.пед.института. -1970. -№1. -С. 55-65.