

NUMERICAL STUDY OF TURBULENT SEPARATED FLOWS IN AXISYMMETRIC
DIFFUSERS BASED ON A TWO-FLUID TURBULENCE MODELShoiev M.A¹, Rakhmankulov S.A¹, Qosimov A.Sh¹, Mandana Bayat²¹Fergana polytechnic institute, Uzbekistan²[Islamic Azad University, Najafabad Branch](#), Iranshoyevmardonbek1@gmail.coms.raxmonqulov@ferpi.uza.sh.qosimov@ferpi.uzmandana.bayat@yahoo.com

Abstract. This paper discusses a numerical study of turbulent flow at $Re = 1.56 \times 10^4$ in an axisymmetric diffuser with half-angle expansion $\alpha = 14^\circ, 18^\circ$ и 90° . The results obtained are compared with known experimental data. The flow at the diffuser inlet is fully developed turbulent. To simulate the flow, a relatively recently developed two-fluid turbulence model in the Comsol Multiphysics software package was used. The paper also presents numerical results of the well-known SST and SA turbulence models, which are included in the Comsol Multiphysics software package. It is shown that the two-fluid turbulence model in the Comsol Multiphysics software package is capable of producing more accurate results than known models. In addition, it demonstrated good convergence and stability.

Key words: Navier–Stokes equations, axisymmetric diffuser, separated flow, two-fluid model.

1. INTRODUCTION

Flow under axisymmetric expansion is one of the most common examples of turbulent flow in practical engineering situations. For example, diffusers with different expansion angles are widely used as a hydraulic element in various technical and technological devices. Flash expansion is also a common geometry in combustion chambers, acting as a flame stabilizer or simply a flash dump diffuser. Experimental and theoretical studies show that the flow in diffusers is quite complex. Under certain conditions, phenomena such as flow separation, reverse flow and increased turbulence can be observed. Moreover, the flow separation in the diffuser strongly depends on its geometry and flow parameters. Therefore, it is important to determine whether boundary layer separation from the surface will occur and to accurately determine the location of flow separation. This phenomenon is important both in theoretical and practical aspects, which is confirmed by many studies. Flow separation occurs when the boundary layer overcomes a positive pressure gradient [1–3]. Finding the break-off point is a difficult task. The work of Chebechi et al. [4] calculated the separation point in incompressible turbulent flows, where four prediction methods were used: the Goldschmid, Stratford, Head and Chebechi-Smith method. The results obtained were subsequently confirmed experimentally. Researchers such as Knob et al [5] have studied the dynamics of boundary layer separation using PIV (Particle Image Velocimetry) and time-resolved biorthogonal decomposition. Gustavsson [6], Yang et al. [7] also experimentally investigated flow separation using PIV, comparing the results with conventional measuring

instruments such as hot wire anemometer and Preston tubes. Works that consider flows passing through a backward-facing step are also devoted to the study of flow separation. These studies are important from the point of view of fundamental fluid mechanics and have many practical applications. All the difficulties associated with the separation and reattachment of a turbulent flow with a positive pressure gradient are observed in this case [8]-[12]. Similar phenomena also occur in sudden expansion channels [13]–[16], which is a common phenomenon in various engineering applications such as combustion chambers, aircraft, pipelines, nuclear reactors, turbomachinery heat exchangers, building fairings, etc. A study of flow separation depending on the diffuser cone angle for axisymmetric expansion was carried out in the work of Chandavari et al. [13]. Also, experimental and numerical studies based on longitudinal flow separation vortices were studied by Thornblom et al. [17].

Stieglmeier, M., Tropea, C., Weiser, N., Nitsche, W. [20-21] conducted an experimental study of flow in an asymmetric diffuser with different diffuser angles ($\alpha=14^\circ$, 18° and 90°), and since then their diffuser has become widely recognized as the standard. Many numerical studies of flow in an asymmetric planar diffuser have been carried out using various turbulence models. Dheeraj Sagar, Akshoy Ranjan Paul, Anuj Jain [22] used the k- ϵ turbulent model. Although this interest in flow has led to a significant amount of experimental and numerical studies of flow fields and their influence on heat and mass transfer coefficients, there is a lack of detailed knowledge about turbulent properties, especially in recirculating regions of flow.

Relatively recently, a work was published in which a new approach to describing turbulence was proposed [23]. In this work, it was demonstrated that a turbulent flow can be represented as a heterogeneous mixture of two fluids with different velocities. Thus, a mathematical model of turbulence was built based on the dynamics of two fluids. The fundamental difference between the two-fluid approach and the Reynolds approach is that the two-fluid approach leads to a closed system of equations, while the Reynolds approach, as is known, leads to an open system of equations. In [24]–[26], solutions to various turbulence problems were obtained based on the two-fluid model. These studies demonstrated the ability of the model to describe thermodynamics in a free turbulent jet, in a rotating flow, and also in flow around a plate. It has been demonstrated that the two-fluid turbulence model is highly accurate, simple to solve engineering problems, and able to adequately describe turbulence anisotropy.

Therefore, the purpose of this work is to validate the two-fluid turbulence model and verify the computational algorithm on a number of simple test problems, axisymmetric expansion with different semi-angles of the diffuser ($\alpha=14^\circ$, 18° and 90°) and compare the results obtained with the results of the known turbulence models SST [27] and SA [28], [29] (which are built into the COMSOL Multiphysics program) and experimental data [15].

2. Physical and mathematical formulation of the problem.

This study focuses on the systematic analysis of separated flow in axisymmetric expansion with different diffuser half-angles, as shown in Figure 1. The geometric parameters of the diffuser are presented in Table 1. The work investigates turbulent flow in a diffuser with half-angle expansion 14° , 18° and 90° at Reynolds number $Re = 1.56 \times 10^4$. The obtained numerical results are compared with experimental data. [15].

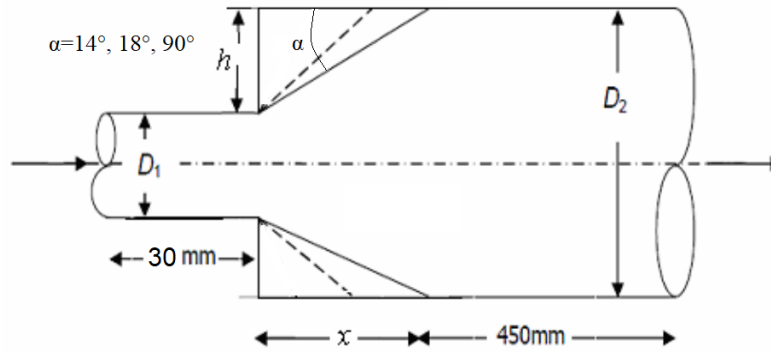


Figure 1. Scheme of the computational domain in an axisymmetric diffuser

Geometric parameters of an axisymmetric diffuser. Table 1:

To describe the movement of a turbulent fluid in an axisymmetric diffuser, as mentioned above, a two-fluid turbulence model was used. The unsteady system of turbulence equations according to the two-fluid model in a cylindrical coordinate system has the following form [17]:

$$\begin{cases}
 \frac{\partial \rho}{\partial \tau} + \frac{\partial \rho V_z}{\partial z} + \frac{\partial \rho V_r}{\partial r} = 0, \\
 \rho \frac{\partial V_z}{\partial \tau} + \rho V_z \frac{\partial V_z}{\partial z} + \rho V_r \frac{\partial V_z}{\partial r} + \frac{\partial p}{\partial z} = \nu \rho \left(\frac{\partial}{\partial r} \left(\frac{\partial V_z}{\partial r} + \frac{\partial V_r}{\partial z} \right) + \frac{1}{r} \left(\frac{\partial V_z}{\partial r} + \frac{\partial V_r}{\partial z} \right) + 2 \frac{\partial^2 V_z}{\partial z^2} \right) - \frac{\partial \rho g_z g_z}{\partial z} - \frac{\partial r \rho g_z g_r}{r \partial r}, \\
 \rho \frac{\partial V_r}{\partial \tau} + \rho V_z \frac{\partial V_r}{\partial z} + \rho V_r \frac{\partial V_r}{\partial r} + \frac{\partial p}{\partial r} = \nu \rho \left(2 \frac{\partial^2 V_r}{\partial r^2} + 2 \frac{\partial V_r}{r \partial r} + \frac{\partial}{\partial z} \left(\frac{\partial V_z}{\partial r} + \frac{\partial V_r}{\partial z} \right) - 2 \frac{V_r}{r^2} \right) - \frac{\partial \rho g_z g_r}{\partial z} - \frac{\partial p r g_r g_r}{r \partial r}, \\
 \rho \frac{\partial g_z}{\partial \tau} + \rho V_z \frac{\partial g_z}{\partial z} + \rho V_r \frac{\partial g_z}{\partial r} = - \left(g_z \rho \frac{\partial V_z}{\partial z} + g_r \rho \frac{\partial V_r}{\partial r} \right) + C_s \rho \left(- \left(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r} \right) g_r \right) + \\
 + \frac{\partial}{\partial z} \left(2 \rho v'_{zz} \frac{\partial g_z}{\partial z} \right) + \frac{\partial}{\partial r} \left(\rho v'_{rz} \left(\frac{\partial g_z}{\partial r} + \frac{\partial g_r}{\partial z} \right) \right) + \frac{\rho v'_{rz}}{r} \left(\frac{\partial g_z}{\partial r} + \frac{\partial g_r}{\partial z} \right) - C_r \rho g_z; \\
 \rho \frac{\partial g_r}{\partial \tau} + \rho V_z \frac{\partial g_r}{\partial z} + \rho V_r \frac{\partial g_r}{\partial r} = - \left(g_z \rho \frac{\partial V_r}{\partial z} + g_r \rho \frac{\partial V_r}{\partial r} \right) + C_s \rho \left(\left(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r} \right) g_z \right) + \\
 + \frac{\partial}{\partial z} \left(\rho v'_{rz} \left(\frac{\partial g_r}{\partial z} + \frac{\partial g_z}{\partial r} \right) \right) + \frac{\partial}{\partial r} \left(2 \rho v'_{rr} \frac{\partial g_r}{\partial r} \right) + \frac{2 \rho v'_{rr}}{r} \frac{\partial g_r}{\partial r} - 2 \rho v'_{rr} \frac{g_r}{r^2} - C_r \rho g_r.
 \end{cases}
 \tag{1}$$

Here

$$v'_{zz} = v'_{rr} = \frac{3}{\text{Re}} + 2 \frac{S}{|\mathbf{def}\bar{\mathbf{U}}|}, \quad v'_{rz} = \frac{3}{\text{Re}} + 2 \frac{|g_z g_r|}{|\mathbf{def}\bar{\mathbf{U}}|},$$

$$|\mathbf{def}\bar{\mathbf{U}}| = \sqrt{2 \left(\left(\frac{\partial V_z}{\partial z} \right)^2 + \left(\frac{\partial V_r}{\partial r} \right)^2 + \left(\frac{V_r}{r} \right)^2 \right) + \left(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} \right)^2}.$$

$$S = \frac{g_z^2 J_z + g_r^2 J_r}{J_z + J_r}, \quad J_z = \left| \frac{\partial g_z}{\partial z} \right|, \quad J_r = \left| \frac{\partial r g_r}{r \partial r} \right|, \quad C_s = 0.2, \quad C_r = C_1 \lambda_{\max} + C_2 \frac{|d \cdot \bar{g}|}{d^2}$$

In the given equations V_z, V_r – respectively the axial and radial components of the averaged flow velocity vector, p – hydrostatic pressure, g_z, g_r – relative axial and radial components of fluid

velocity, ν - molar kinematic viscosity, $\nu'_{zz}, \nu'_{rr}, \nu'_{zr}$ - effective molar viscosities, d - closest distance to a solid wall, λ_{\max} - largest root of the characteristic equation.

$$\det(A - \lambda E) = 0, \tag{2}$$

where is the matrix $A = \begin{vmatrix} -\frac{\partial V_z}{\partial z} & -\frac{\partial V_z}{\partial r} - C_s \left(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r} \right) \\ -\frac{\partial V_r}{\partial z} + C_s \left(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r} \right) & -\frac{\partial V_r}{\partial r} \end{vmatrix}$

The largest root of the characteristic equation is equal to $D = \frac{\partial V_z}{\partial r} \frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial z} \frac{\partial V_r}{\partial r} + C_s(1 - C_s) \left(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r} \right)^2$,

$$\lambda_{\max} = \sqrt{D}, \text{ если } D > 0,$$

$$\lambda_{\max} = 0, \text{ если } D < 0.$$

Constant coefficients are equal $C_1 = 0.7825, C_2 = 0.306$.

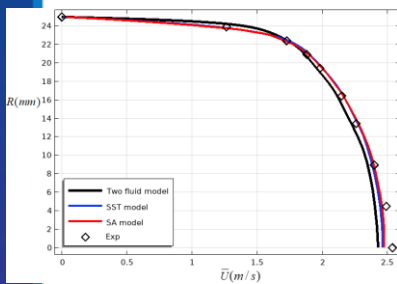
3. Solution method

COMSOL Multiphysics provides a wide range of solvers to solve a variety of physics problems. The choice of a particular solver depends on the type of physics being modeled, the complexity of the problem, the required accuracy, and the available computing resources. To solve the equations of the two-fluid turbulent model, the Fully Coupled method was chosen using the PARDISO direct solver algorithm. In this case, Newton's iterative method with a damping coefficient of 0.1 was used. The iteration process for the stated problem continued up to 250 iterations. The tolerance factor was set to 1, and the residual factor was set to 1000. These parameters play an important role in the solution process and allow us to achieve the necessary balance between calculation accuracy and computational efficiency.

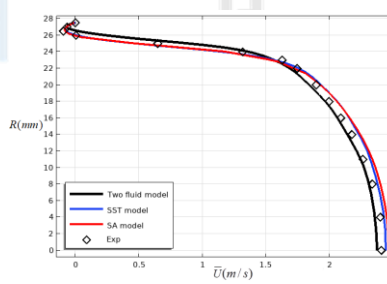
Standard COMSOL Multiphysics solvers were used for the standard SST and SA turbulence models.

4. The discussion of the results

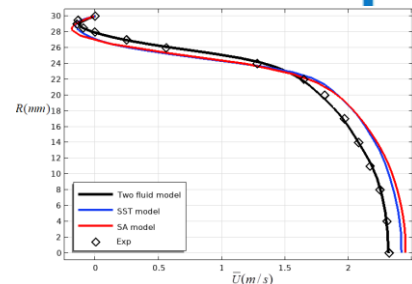
In Fig. 4-6 show graphs comparing calculated and experimental data for an axisymmetric diffuser with $\alpha=14^\circ$. For comparison, these figures also show the numerical results of the SST and SA turbulence models. Figure 4 shows the profiles of the axial U-component of the velocity; Fig. 5 profiles of the radial V-velocity component and Fig. 6 profiles of flow pulsation $u'v'$ in various sections.



x=0 (mm)



x=10 (mm)



x=20 (mm)

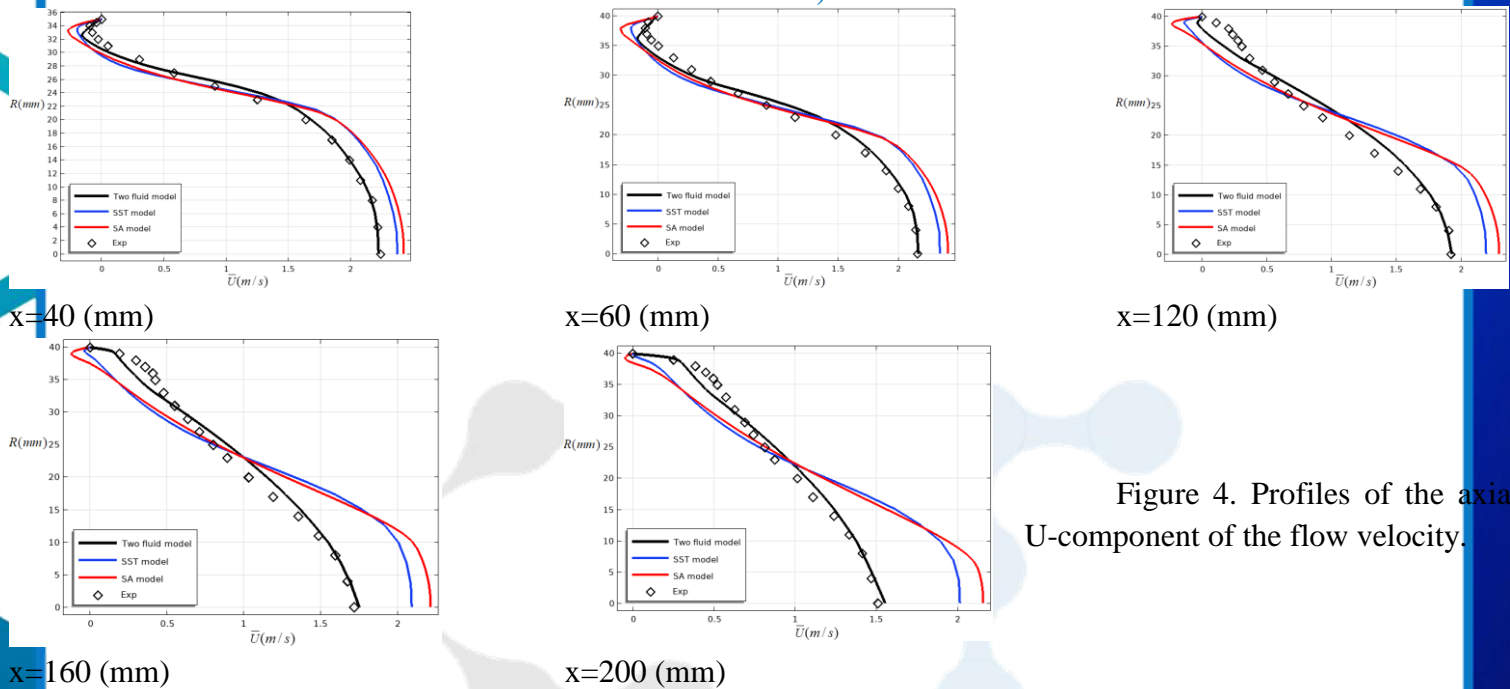


Figure 4. Profiles of the axial U-component of the flow velocity.

From Figure 4 you can see that for the initial sections at $x=0$ (mm) and $x=10$ (mm) the results of all models are approximately the same. However, starting from $x=20$ (mm) and in all subsequent sections, the results of the SST and SA models diverge significantly from the experiment, while the results of the two-fluid model show good agreement with the experimental data.

5.CONCLUSION

As a result of the analysis of the presented graphs and data, we can conclude that when studying the flow in expanding channels, the behavior of the models differs depending on the distance from the initial expansion section. In the initial expansion region, all models show similar results. However, as we move away from this point, the two-fluid model turns out to be more adequate, providing a more accurate reproduction of experimental data. Thus, in this study, the two-fluid model stands out for its efficiency and accuracy in describing turbulent flow in an expanding pipe.

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